

*Instruction Manual*  
Vibrating Wire  
**Stressmeter**  
4300 Series (EX, BX, NX)



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## 1. Specifications

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<b>General Specifications</b>	<b>EX</b>	<b>BX</b>	<b>NX</b>
<b>Nominal Range Mpa (psi)</b>	35 - 100 (5000 -	35 - 100 (5000 -	35 - 100 (5000 -
<b>Resolution KPa (psi)</b>	2 - 15 (0.3 - 2)	10 - 30 (1.5 - 4)	35 - 140 (5 - 20)
<b>Accuracy <math>\pm</math> <sup>2</sup></b>	20 %	20 %	20 %
<b>Operating Temperature</b>	-30 to +90	-30 to +90	-30 to +90
<b>Thermal Zero Shift %</b>	0.02	0.04	0.04
<b>Resonant Frequency</b>	3000 - 5000	2000 - 3500	1500 - 2500
<b>Length mm (inches)</b>	44 (1.75)	70 (2.75)	76 (3.0)
<b>Outer Diameter mm</b>	29 (1.125)	48 (1.875)	64 (2.50)
<b>Inner Diameter mm</b>	13 (0.5)	22 (0.875)	32 (1.25)
<b>Weight kgm (lbs.)</b>	0.45 (1)	0.9 (2)	1.4 (3)
<b>Borehole Diameter mm</b>	38 (1.485)	60 (2.36)	76 (2.98)
<b>Gage Materials</b>	17-4 SS, 304 SS	17-4 SS, 304 SS	17-4 SS, 304 SS
<b>Cable</b>	2 conductor or 4 conductor shielded 22 gage. PVC jacket,		

<sup>1</sup> Depends on rock modulus

<sup>2</sup> Accuracy depends to a large extent on the roughness of the borehole walls; on the degree to which the platens bed into the surrounding material thus increasing the area of contact and on the gage stiffness; and also on the accuracy with which the host rock elastic constants are known.

<sup>3</sup> High temperature versions are available (-20 °C to 200 °C)

## 2. Theory of Operation

Geokon vibrating wire stressmeters are designed primarily for long-term measurements of stress changes in rock, by utilizing a vibrating wire transducer to measure the deformation of a thick-walled steel ring preloaded into a borehole by a wedge and platen assembly as shown in figure 1.

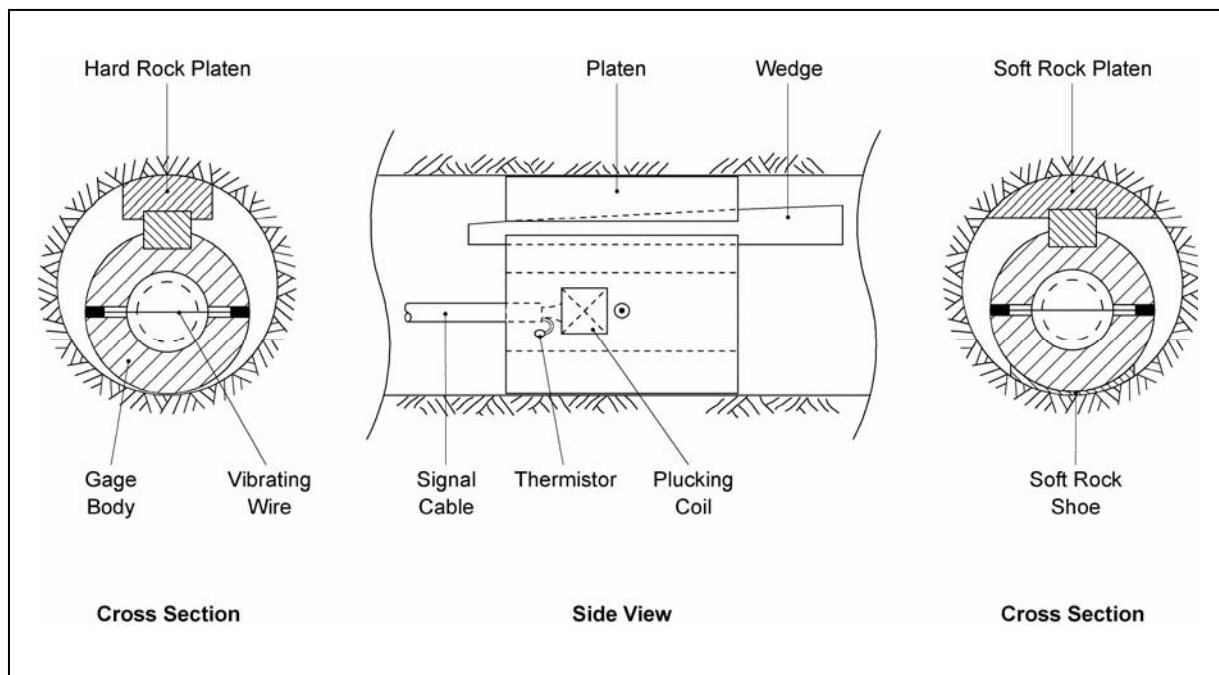


Figure 1. Vibrating Wire Stressmeter.

In use, changing rock stresses impose changing loads on the gage body causing the body to deflect, and this deflection is noted as a change in tension and resonant frequency of vibration of the vibrating wire element. The square of the vibration frequency is directly proportional to the change in diameter of the gage and, by calibration, to the change in stress in the rock.

The actual calibration of the gage depends upon many factors including the host rock elastic constants, the pre-stress applied during installation, the orientation of the stressmeter with respect to the principal rock stress direction and platen contact area. Thus the accuracy of the gage reading is largely indeterminate and the indicated stress magnitude can only be approximate.

A coil and magnet assembly located close to the wire is used both to excite the wire and sense the resultant frequency of vibration. When the gage is connected, a pulse of varying frequency is applied to the coil and magnet assembly, and this causes the wire to vibrate at its resonant

frequency. The wire continues to vibrate, and a signal, at the gage frequency, is induced in the pickup coil and transmitted to the readout box where it is conditioned and displayed.

In theory, where the effective modulus of the stressmeter (approximately 28Gpa ( $4 \times 10^6$  PSI)) is more than two times the modulus of the host rock, conversion of the readings to changes in stress does not require an accurate knowledge of the rock modulus, and this is the reason for using the term stressmeter for this device. However, in most rocks, and especially in harder rocks, the modulus must be known to improve the accuracy of the stress measurements, and calibration curves provided herein give sensitivity factors for materials of different moduli. It should be noted that as the rock modulus changes by a factor of 10, the gage factor changes only by a factor of 2.

The stressmeter is a uniaxial device, and to completely evaluate stress changes in a given plane, three stressmeters, installed at  $0^\circ$ ,  $45^\circ$ , and  $90^\circ$  orientations, are required.

The gage wire in the Model 4300 Series stressmeters runs perpendicular to the direction in which the gage body is loaded in an effort to minimize the effects of point loading, off center loading, etc. This gives the gage a very high range and, since as the load increases the wire gets tighter, the wire never goes slack.

Gage installation is accomplished by driving a wedge between the gage body and the platen, which contacts the borehole walls. Preloading to desired levels is accomplished by further driving of the wedge with the setting tool. In soft rocks a soft rock platen and soft rock shoe are used to increase the area of contact.

The gage is constructed of corrosion resistant materials and should have an indefinite lifetime under even the most severe conditions.

### 3. Installation \_\_\_\_\_

#### **3.1 Borehole Requirements**

Stressmeters are designed to be used in smooth-walled diamond drill holes. Stressmeters can be installed in percussively drilled holes and drag-bit drilled holes, provided that care is taken to get the proper hole diameter with a smooth wall. If the walls are rough the gage response (calibration) can be radically affected.

The Model 4300EX Stressmeter is designed for use in EX diamond drill holes 38mm (1.5”), and the hole can range in diameter from 37mm (1.45”) to 39mm (1.55”) when using the standard wedge and platen assembly.

The Model 4300BX Stressmeter is designed for use in BX diamond drill holes 60mm (2.36”), and the hole can range in diameter from 58.5mm (2.30”) to 62mm (2.44”) when using the standard wedge and platen assembly.

The Model 4300NX Stressmeter is designed for use in NX diamond drill holes 76mm (2.98”), and the hole can range in diameter from 74mm (2.91”) to 78mm (3.07”) when using the standard wedge and platen assembly. Oversize platens are available for over-size boreholes (consult factory).

After drilling, the hole should be thoroughly cleaned by washing out with water or blowing out with compressed air. The borehole diameter should then be checked with the GO-NO-GO gages supplied with the installation tool. If the borehole checks out the installation can proceed.

### **3.2 Preliminary Checks**

Upon receipt of the stressmeter the zero reading should be checked and noted along with the temperature if a thermistor is included in the gage. Gage connections are normally red to red and black to black, although with the GK-401 Readout Box these can be reversed without changing the readings. Zero readings at the site should coincide with the factory readings within a few digits after corrections for temperature are made.

### **3.3 Attaching the Wedge/Platen Assembly**

The wedge/platen assemblies are shipped separately. They are held together by a nylon screw and nut. Remove the nut and then use the nylon screw to attach the wedge/platen assembly to the Stressmeter Body. Orient the wedge so that the narrow end is facing in the same direction as the cable, (see Figure 2). Tighten the nylon screw into the threaded hole in the body. Do not over-tighten – the screw is made of nylon so that it can be sheared more easily. (**Note:** The BX size stressmeter and the NX size stress meter both use the same wedge. However, there are two holes in the wedge. The one nearest the tip is for the BX size, the one farthest from the tip is for the NX size).

### 3.4 Setting the Stressmeter (Recoverable Type)

Mount the stressmeter on the setting tool by pushing the nylon threaded pieces into the matching holes in the setting tool head. Feed the gage leads through the slot in the setting head. (See figure 2 on page 5).

Connect the first section of  $\frac{1}{4}$ " rod to the yoke attached to the thin end of the wedge. Note that the first section of rod has one end with a  $\frac{1}{4}$ -20 **Left Hand Thread** on it. This will connect to the left-hand thread in the yoke.

Attach the first section of the  $\frac{3}{4}$ " positioning rod to the back of the setting tool head. Push the stressmeter into the hole using the positioning rod. The buttons on the setting rod connectors indicated the orientation of the wedge/ platen assembly. Thus for making measurements in a vertical direction keep the buttons to the top of the rod, etc. As the  $\frac{3}{4}$ " positioning rod is pushed into the hole, add new sections of both  $\frac{3}{4}$ " and  $\frac{1}{4}$ " rod until the desired depth has been reached. **It is advisable to wear gloves during this procedure to protect the thumb while depressing the buttons on the  $\frac{3}{4}$  inch rods.** Now slide the slide hammer over the last section of  $\frac{1}{4}$ " rod and then thread the anvil block onto the outer end of the  $\frac{1}{4}$ " rod. Connect the GK-403 or GK-401 readout box to the lead wires and switch to position "F" for EX size or "B" for BX and NX size. Take initial readings.

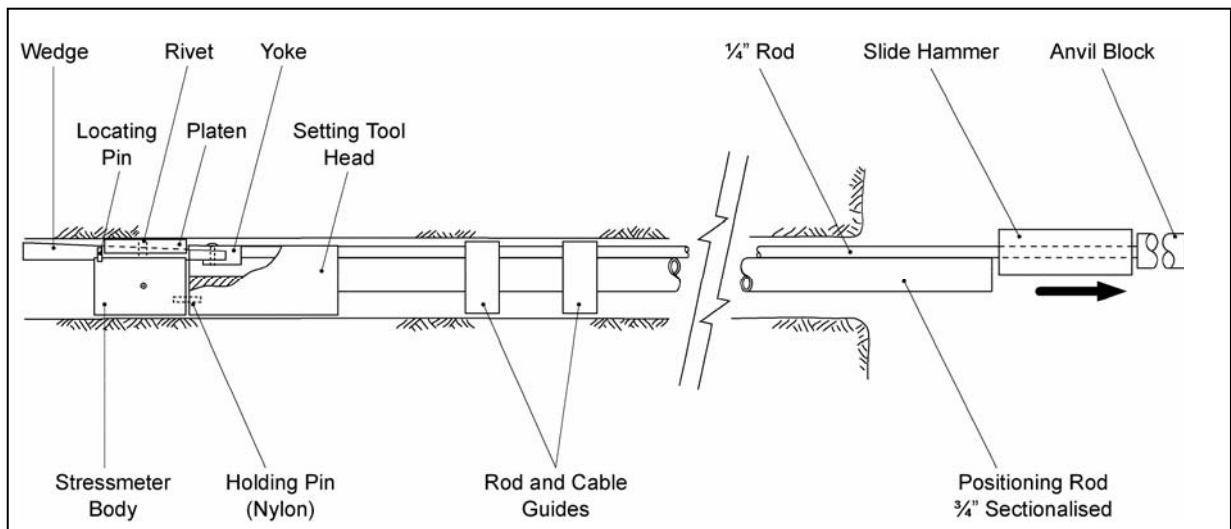


Figure 2. Vibrating Wire Stressmeter installation tool assembly.

Holding the positioning rod firmly at its correct depth and orientation, slide the slide hammer back up the ¼” rod, then slide it quickly back to the anvil striking it a sharp firm blow. This will shear the rivet holding the wedge to the platen and will pull the wedge into the platen thereby expanding it against the wall of the borehole.

After the first blow, take another reading on the GK-403 or GK-401 and observe the change in reading. The recommended preloads are as follows: for the EX size a reading change of 2000 digits on channel F, for the BX size a reading change of 400 digits on channel B, for the NX size a reading change of 250 digits on channel B.

Use as many blows of the hammer as is necessary to achieve this reading **but stop hammering if successive blows produce little or no change of reading**. When the target reading has been achieved or if successive hammer blows produce little or no change, disconnect the ¼” rod from the wedge yoke by turning clockwise (remember, left hand thread). Remove the ¼” rod from the hole, then disengage the setting tool from the stressmeter by pulling on it.

For multiple installation of gages in a single hole, route the lead wires from deeper gages in the recess in the side of the setting tool head. Maintain tension on these wires, as subsequent gages are pushed into the hole.

If necessary, after setting the gages and obtaining the final readings, push the leads back into the borehole and seal the borehole using an expandable rockbolt anchor or a short bolt. This will discourage vandalism if this is a problem.

Note that the stressmeter initial readings will probably diminish slightly over the first day or two as the stressmeter beds firmly into place.

### **3.5 Recovering the Stressmeter**

After tests, the stressmeter can be removed from the borehole by using the setting tool. Only the larger setting rods are required along with the setting tool head, which is used to strike the outer tip of the wedge. This will drive the wedge out from under the platen and allow the stressmeter to be pulled from the hole using the electrical cable. Make sure that the setting head is twisted so that the flat part of the front face lies opposite the wedge. The entire stressmeter can sometimes be recovered in this way i.e., the wedge, platen and stressmeter

body. To reuse these components will require a new nylon screw. (A few spare nylon screws are included in each shipment). However, there is a good chance that the wedge and platen may dislodge in the borehole and be lost so it will be advisable to carry spares of these also. (**Note:** The BX size stressmeter and the NX size stress meter both use the same wedge. However, there are two holes in the wedge. The one nearest the tip is for the BX size, the one farthest from the tip is for the NX size).

### **3.6 Splicing and Junction Boxes**

Because vibrating wire readout is frequency rather than current or voltage, slight variations in cable resistance have no ill effect on gage readings, and therefore splicing of cables presents no problem for installations and, in many cases, can make the job easier. Splicing boxes allow for the use of multiconductor cables and enable the gages to be set with a minimum of cable exposed in working areas. When properly made, splices are equal to the cable itself in strength and electrical properties. Terminal boxes with switches or plug-in connections are available for termination of multiple lead wires at a single readout location. Splicing materials, junction and splicing boxes and terminal boxes, along with instructions, are available from Geokon.

## 4. Taking Readings \_\_\_\_\_

### **4.1 Operation of the GK-403**

The GK-403 Readout Box provides the necessary excitation and signal conditioning for the Model 4300 Series Stressmeters. To take readings, the box is connected to the gage by a jumper with either clip leads or, in the case of a terminal station, with a connector.

1. Turn the display selector to position "F" for EX size or position "B" for BX and NX sizes.
2. Turn the unit on and a reading will appear in the front display window. The last digit may fluctuate by several digits and this is explained below.
3. Zeros in the display indicate either a faulty connector, a damaged gage or high levels of electrical noise. Connect the ground lead to the cable shield (in this last case) and if the signal does not appear, trouble shooting is required (see Section 6).
4. The unit will automatically turn off after approximately 4 minutes to conserve power.

As noted above, the last digit in the display will very often fluctuate by several digits, and this should not be seen as abnormal operation. In the case of the stressmeter, the vibrating wire is very short and the signals are not as pure as those of other gages. This, coupled with the fact that

we are presenting frequency squared, causes some instability, which shows up in the least significant digit. This is not to say that the readings are not accurate; it simply means that the period of vibration changes very slightly from one pluck to the next. In most cases the displayed numbers should be rounded to the next least significant digit. For very stable stressmeters the last digit can give very valuable information on very small stress changes, and for that reason the numbers are not rounded off electronically.

#### 4.2 Operation of the GK404 Readout Box

The GK404 is a palm sized readout box which displays the Vibrating wire value and the temperature in degrees centigrade.

The GK-404 Vibrating Wire Readout arrives with a patch cord for connecting to the vibrating wire gages. One end will consist of a 5-pin plug for connecting to the respective socket on the bottom of the GK-404 enclosure. The other end will consist of 5 leads terminated with alligator clips. Note the colors of the alligator clips are red, black, green, white and blue. The colors represent the positive vibrating wire gage lead (red), negative vibrating wire gage lead (black), positive thermistor lead (green), negative thermistor lead (white) and transducer cable drain wire (blue). The clips should be connected to their respectively colored leads from the vibrating wire gage cable.

For EX size use the **POS** (Position) button to select position **F** and the **MODE** button to select **Dg** (digits).

For BX and NX sizes use the **POS** (Position) button to select position **B** and the **MODE** button to select **Dg** (digits).

Other functions can be selected as described in the GK404 Manual.

The GK-404 will continue to take measurements and display the readings until the OFF button is pushed, or if enabled, when the automatic Power-Off timer shuts the GK-404 off.

The GK-404 continuously monitors the status of the (2) 1.5V AA cells, and when their combined voltage drops to 2V, the message **Batteries Low** is displayed on the screen. A fresh set of 1.5V AA batteries should be installed at this point

## 5. Data Reduction

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The GK-403 Readout excites the gage and measures the period of 255 cycles (or less) of gage

vibration, using a 6.144 MHz quartz oscillator, and displays the period to a resolution of 0.1 microseconds in position “A”. Positions “F” and “B” are used for stressmeters, the processor converts the period readings to units of frequency squared which is proportional to wire strain, gage deflection and applied stress. A reading of 10,000 on channel “F” corresponds to a period of 316.2 microseconds on channel “A”.

To obtain the change in stress at any given time the following equation applies:

$$\sigma = (R_1 - R_0) G, \text{ where}$$

$\sigma$  = stress in psi

$R_0$  = initial reading at zero stress (“B” or “F”, GK-401 or GK-403)

$R_1$  = reading at subsequent stress (“B” or “F”, GK-401 or GK-403)

$G$  = sensitivity factor taken from Fig 3, 4 or 5

*For example:*

<b>EX</b>	<b>BX</b>	<b>NX</b>
Readout initial display = 10,000 Subsequent display = 12,000	Readout initial display = 4,000 Subsequent display = 5,000	Readout initial display = 2,500 Subsequent display = 3,000
$\sigma = (R_1 - R_0) G$	$\sigma = (R_1 - R_0) G$	$\sigma = (R_1 - R_0) G$
$\sigma = (12,000 - 10,000) 0.50$	$\sigma = (5,000 - 4,000) 2.5$	$\sigma = (3,000 - 2,500) 6.0$
$\sigma = 1,000 \text{ psi}$	$\sigma = 2500 \text{ psi}$	$\sigma = 3,000 \text{ psi}$

### 5.1 Gage Sensitivity Factors

Figures 3, 4 or 5 are used for determining the stress sensitivity or gage factor for rocks of different moduli. Sensitivity factors are based on experimental data conducted on rock samples and can only serve as a guide.

For more accurate determinations of stress sensitivity, calibrations must be performed in samples of the rock being monitored.

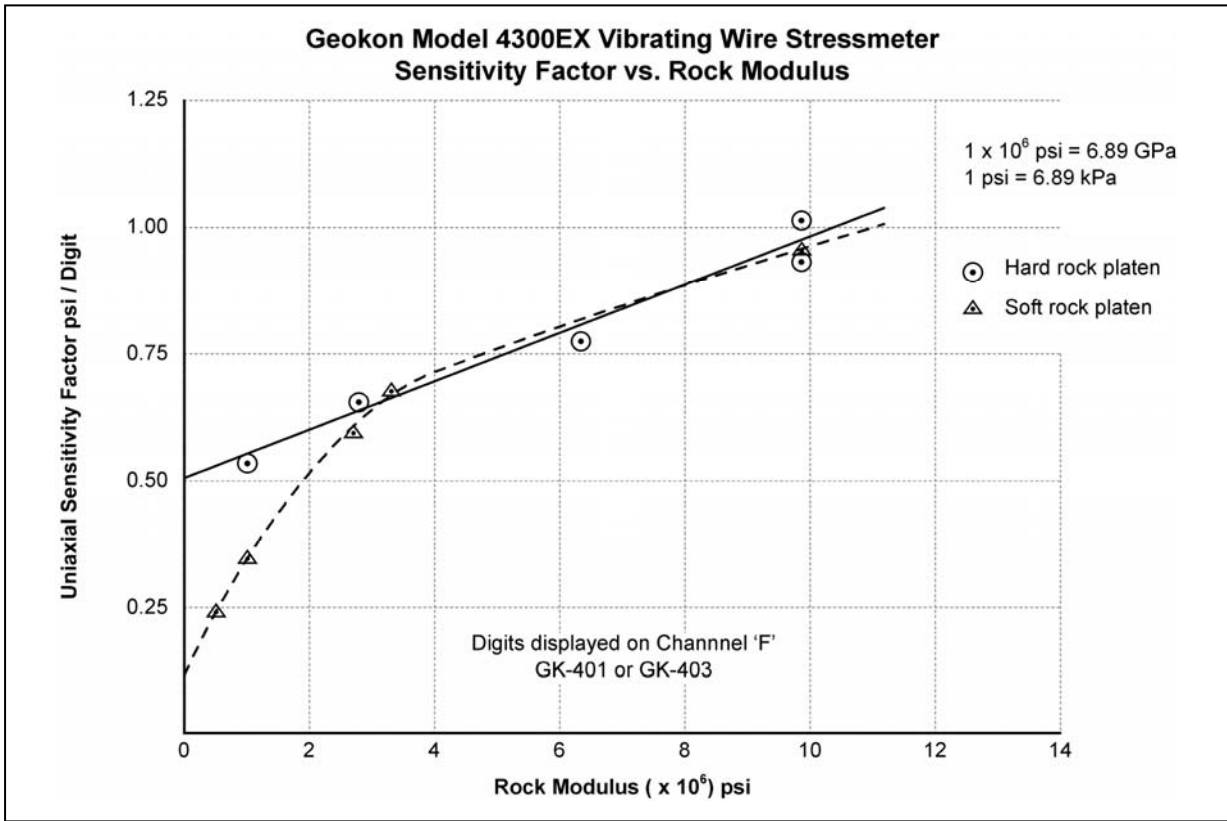


Figure 3. Model 4300EX Vibrating Wire Stressmeter Sensitivity Factor VS. Rock Modulus

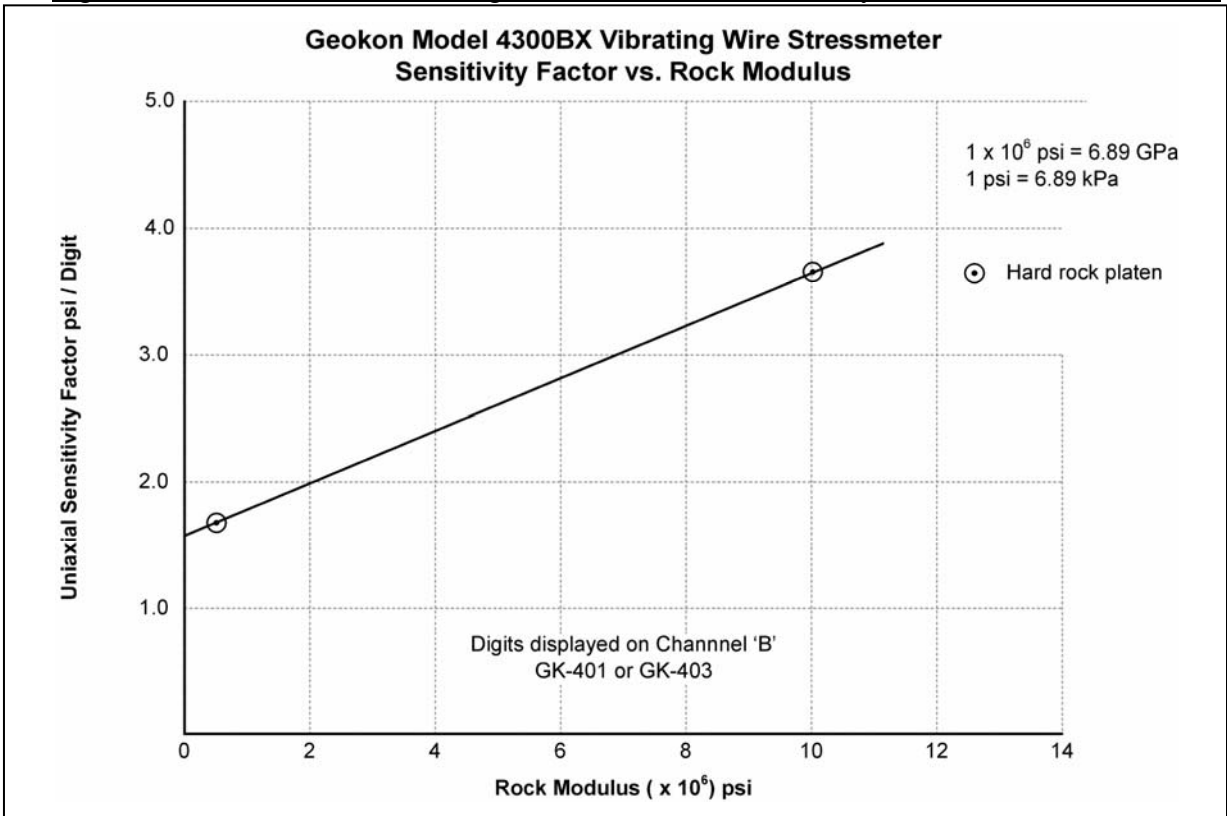


Figure 4. Model 4300BX Vibrating Wire Stressmeter Sensitivity Factor VS. Rock Modulus

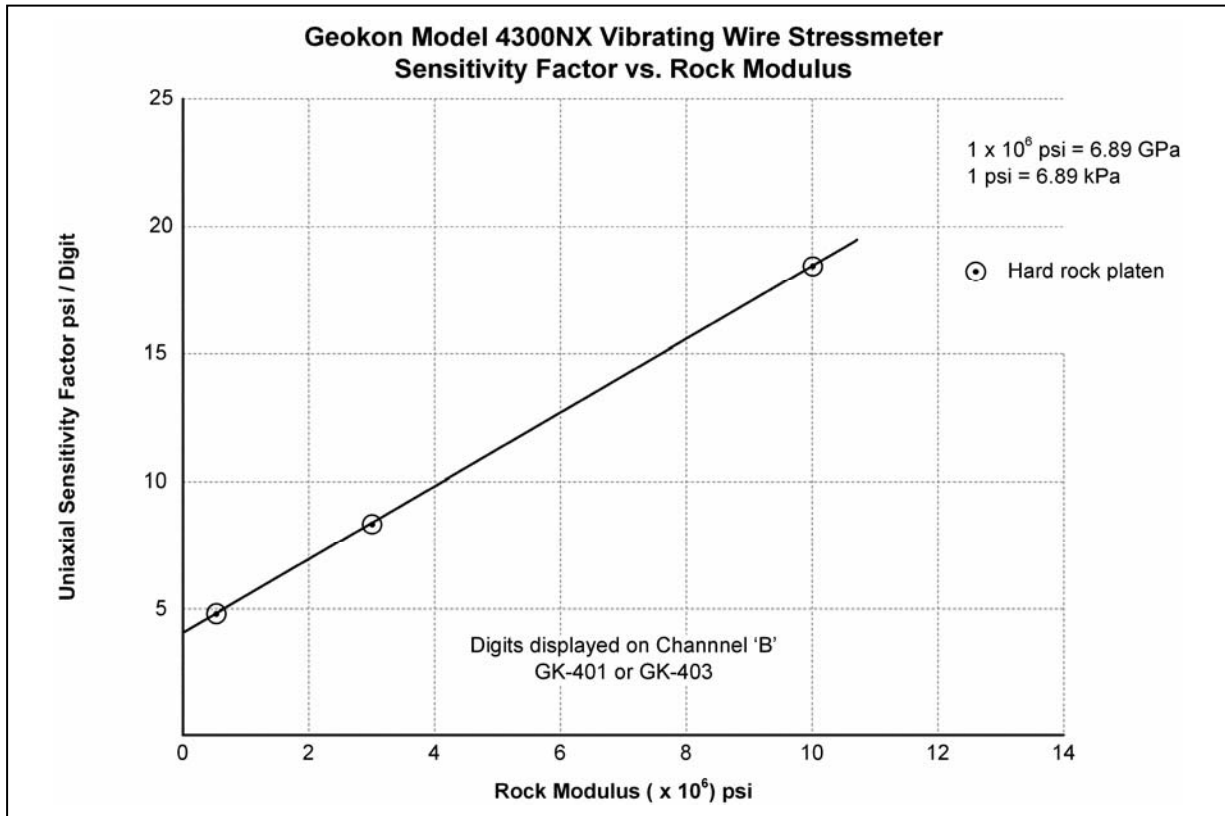


Figure 5. Model 4300NX Vibrating Wire Stressmeter Sensitivity Factor vs. Rock Modulus

## 5.2 Corrections for Temperature Changes

Because of the materials used in construction of the stressmeter the device is affected by changes in ambient temperature. Since these gages are normally installed underground in constant temperature environments corrections are not normally applied.

However, corrections can be made and transducers can be equipped with thermistors for temperature measurement. The temperature correction factor for the gage is 2 readout box units/°C, indicating as apparent decrease in rock stress for a temperature rise.

Stress correction for temperature is:

$\sigma_T = (R_1 - R_0) G + (T_1 - T_0) 2G$ , where

$\sigma_T$  = the stress change corrected for temperature

$R_0$  = initial reading

$R_1$  = subsequent reading

$T_0$  = initial temperature °C

$T_1$  = subsequent temperature °C

$G$  = sensitivity factor

It should be noted that this temperature correction factor is for a gage in a free field with no restraints. In a field condition where the gage is firmly placed in a borehole the gage temperature sensitivity is also dependent on the gage/rock interactions, and these relationships are very complex and beyond the scope of this manual. Calibration would be required for accurate determination of the thermal characteristics of the gage.

### 5.3 Environmental Factors

Since the purpose of the stressmeter installation is to monitor site conditions, factors that may affect these conditions should always be observed and recorded. Seemingly minor effects may have a real influence on the behavior of the structure being monitored and may give an early indication of potential problems. Some of these factors include, but are not limited to: blasting, rainfall, tidal levels, excavation and fill levels and sequences, traffic, temperature and barometric changes, changes in personnel, nearby construction activities, seasonal changes, etc.

## 6. Trouble Shooting \_\_\_\_\_

Maintenance and trouble shooting of vibrating wire stressmeters is confined to periodic checks of cable connections and maintenance of terminals. The transducers themselves are sealed and cannot be opened for inspection. The setting rods should be kept clean and the button mechanisms kept lightly oiled.

If a unit fails to read, the following steps should be taken:

1. Check the coil resistance. Nominal coil resistance is  $90 \Omega \pm 5$  for EX;  $180 \Omega \pm 5$  for BX and NX, plus cable resistance (22 gage copper = approximately  $20 \Omega$  per 1000 feet).

- a. If the resistance is high or infinite a cut cable must be suspected.
  - b. If the resistance is low or near zero a short must be suspected.
  - c. If resistances are within the nominal range and no reading is obtained, the transducer is suspect and the factory should be consulted.
  - d. If all resistances are within nominal and no readings are obtainable on any transducer, the readout is suspect and the factory should be consulted.
2. If cuts or shorts are located, the cable may be spliced in accordance with recommended procedures.
  3. If readings are unstable try connecting the ground clip on the readout box to the cable shield.





20.70 K	-14	2872	26	603.3	66	171.4	106	61.1	146
19.58 K	-13	2750	27	582.6	67	166.7	107	59.6	147
18.52 K	-12	2633	28	562.8	68	162.0	108	58.3	148
17.53 K	-11	2523	29	543.7	69	157.6	109	56.8	149
								55.6	150

## Appendix 1: Biaxial Stress Changes

The relationship between the radial deformation of a borehole,  $U$ , and two principle stresses in the plane of a borehole has been given by Hast (1958) and Merrill and Peterson (1961).

For Plane Stress:

$$U = d/E_r [(\sigma_1 + \sigma_2) + 2 (\sigma_1 - \sigma_2) \cos 2 \theta]$$

Where:  $\sigma_1$  and  $\sigma_2$  are the principle stresses in the plane of the borehole.

$\theta$  is the angle measured counterclockwise from the direction of  $\sigma_1$ .

$d$  is the diameter of the borehole.

$E_r$  is the Youngs modulus of the rock

If we assume that the stress measured across the stressmeter is proportional to the radial deformation that would have occurred in this direction if the stressmeter had not been there, then the term  $d/E_r$  can be replaced by one reflecting the relationship between the rock modulus and the gage modulus. Hast (1958) has shown this to be applicable for a uniaxial stressmeter.

For the measurement of stress  $\sigma_R$  in any direction  $\theta$  the following applies:

$$\sigma_R = 1/3 (\sigma_1 + \sigma_2) + 2/3 (\sigma_1 - \sigma_2) \cos 2 \theta$$

( $\theta$  measured counterclockwise from  $\sigma_1$ )

Using this relationship and three uniaxial stress change measurements at  $45^\circ$  to each other, the secondary principle stresses  $\sigma_1$  and  $\sigma_2$  and the angle  $\theta$  are given by:

$$\sigma_1 = 3/2 a + 3/4 b$$

$$\sigma_2 = 3/2 a - 3/4 b$$

$$\theta = 1/2 \sin^{-1} ((a - \sigma_{45})/b)$$

where:  $a = \sigma_0 + \sigma_{90} / 2$

$$b = [(\sigma_{45} - a)^2 + (\sigma_0 - a)^2]^{1/2}$$

To determine the  $\theta$  angles, you have to determine what quadrant the angle lies in. The inequalities to do this are as follows:

$$\text{If } \sigma_{45} \leq a \text{ and } \sigma_0 \geq 90, \text{ then } 0 \leq \theta \leq 45^\circ$$

$$\text{If } \sigma_{45} \leq a \text{ and } \sigma_0 \leq 90, \text{ then } 45^\circ \leq \theta \leq 90^\circ$$

$$\text{If } \sigma_{45} \geq a \text{ and } \sigma_0 \leq 90, \text{ then } 90^\circ \leq \theta \leq 135^\circ$$

$$\text{If } \sigma_{45} \geq a \text{ and } \sigma_0 \geq 90, \text{ then } 135^\circ \leq \theta \leq 180^\circ$$

Note:  $\theta$  is measured clockwise for  $\sigma_0$  (this is same as counter-clockwise for  $\sigma_1$ ).

**Example:**

Three gages are set in borehole. The first is at  $0^\circ$  ( $\sigma_0$ ), the second at  $45^\circ$  ( $\sigma_{45}$ ) and the third at  $90^\circ$  ( $\sigma_{90}$ ), measured counter-clockwise from 0. The uniaxial stress changes for each gage are determined by the reading change times the calibration factor.

Substitute the constants into the equations to obtain the magnitude of the changes of the two secondary principal stresses,  $\sigma_1$  relative to  $0^\circ$ .

Stress Changes:

$$\text{Gage 1, } \sigma_0 = 600 \text{ psi}$$

$$\text{Gage 2 } \sigma_{45} = 800 \text{ psi}$$

$$\text{Gage 3 } \sigma_{90} = 300 \text{ psi}$$

Calculate the values for constants, a and b:

$$a = \sigma_0 + \sigma_{90}/2 = 600 + 300/2 = 450$$

$$b = [(\sigma_{45} - a)^2 + (\sigma_0 - a)^2]^{1/2} = [(800-450)^2 + (600-450)^2]^{1/2} = 380.79$$

$$\sigma_1 = 3/2a + 3/4b = 3 \times 450/2 + 3 \times 380.79/4 = 960.59 \text{ psi}$$

$$\sigma_2 = 3/2a - 3/4b = 3 \times 450/2 - 3 \times 380.79/4 = 389.41 \text{ psi}$$

$$\sin 2\theta = -0.92$$

$$\theta = 33.40^\circ$$

$\sigma_1$  Direction: since  $\sigma_{45} > a$  and  $\sigma_0 > \sigma_{90}$ , then  $135 < \theta < 180^\circ$ . Therefore,  $\theta = 180 - 33.40 =$

146.6°. This is measured clockwise from  $\sigma_0$ .

References

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Hast, N.; THE MEASUREMENT OF ROCK PRESSURE IN MINES;

Sveriges Geologiska Undersokning, Arsbok 52, Series C, 3. 1958.

Merrill, R.H. and Peterson, J.R.; DEFORMATION OF A BORE HOLE IN ROCK;

U.S. Bureau of Mines, RI 5881.