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Instruction Manual

Model 4350BX

Biaxial Stressmeter

(Doc Rev D, 6/10)

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LIST of FIGURES, TABLES and EQUATIONS

1. Theory of Operation

The Standard Geokon Model 4350 Biaxial Stressmeter is designed to measure changes in stress in hard rocks, rock-salt, potash, concrete, ice and other elastic and viscoelastic materials. The sensor consists of a thick walled steel cylinder, which is grouted in a borehole or embedded in the material to be investigated. Three or six vibrating wire sensors measure the radial deformation of the cylinder and, using theoretically derived equations, the associated stress changes can be determined. The three or six measurements are made in the plane perpendicular to the borehole at 60º intervals allowing determination of the changes in the biaxial stress field in the material around the sensor. Two sensors can also be incorporated into the biaxial stressmeter for the measurement of longitudinal deformations of the stressmeter. This allows the correction of effects due to variations in the stress directed along the borehole. Also two vibrating wire temperature sensors are connected to compensate for temperature variations. (The ice gage variant, USS+ACE CRREL style, uses only three radial gages and two thermistors to measure the temperature)

The sensing elements are vibrating wire strain transducers, which are anchored across the diameter of the cylinder and are used to precisely measure the deformation of the cylinder. Coil and magnet assemblies, located close to the wires, are used both to excite the wires and sense the resultant frequency of vibration. When a gage is connected, a pulse of varying frequency is applied to the coil and magnet assembly, and this causes the wire to vibrate at its resonant frequency. The wire continues to vibrate and an electrical current, at the gage frequency, is induced in the pickup coil and transmitted to the readout box where it is measured and displayed.

The term, stressmeter, is used for this device because its effective modulus is deliberately kept high (13 x 10^6 psi) so that variations in the modulus of the surrounding medium cause only small variations in the calibration of the gage. For example, if the modulus of the medium varies by a factor of 10 the stressmeter calibration varies by a factor of 2 only.

Installation of the gage is accomplished by inserting the gage into a grout-filled borehole using a setting tool and self-aligning setting rods. When the proper location and orientation is achieved, a cable connected to an anchor release pin is pulled, freezing the gage in place and allowing the setting equipment to be removed. Special expansive grouts are normally used to insure that the gage is in complete contact with the surrounding rock. Theoretical computations, given in the Appendix, assume that the stressmeter is perfectly bonded to the surrounding medium.

2. Installation

2.1 Preliminary Tests

Upon receipt of the stressmeter and prior to installation, the zero readings should be checked and noted. Wiring diagrams are shown in Appendix 3.

The gage should be allowed to come to ambient temperature before zero readings are recorded, and this can take as long as a few hours because of the great mass of the gage. The readings should check very closely with factory zero readings.

2.2 Borehole Requirements

The Model 4350 Stressmeter is designed for grouting inside a BX (2.36 inch) diamond drill boreholes. The stressmeter body is 2.250 inch in diameter and centering buttons extend to a 2.32 inch circle. A test plug 2.250 inches in diameter can be pushed down the hole to check for a minimum diameter, and the centering buttons can be filed off if they protrude too far. If the hole is oversize the buttons can be fitted with a screw or some other device to enlarge the centering circle. A larger annulus means a thicker layer of grout and this will have some influence on gage output.

For horizontal holes, the hole should be drilled slightly downwards, enough to ensure that the grout will not drain from the borehole and that the stressmeter will be fully surrounded with grout. Upward directed boreholes are not recommended.

After drilling, the hole should be thoroughly cleaned by washing out with water or blowing out with compressed air.

If the borehole diameter checks out, the installation can proceed.

2.3 Grouting

To ensure that the gage is in intimate contact with the surrounding rock, an expansive grout should be used. A grout that has been used with good success is manufactured by the U.S. Grout Company called:

5 Star Micro Aggregate
Special Grout 400

This is a microfine cable anchor grout with very high strength.

The hole should be filled with grout from the bottom. After removing the grout pipe the stressmeter can be installed by pushing it into the grout until it reaches the desired location.

Note: In highly fractured rocks, the borehole may require grouting and re-drilling to ensure that the gage is completely grouted. In ice the gage is simply allowed to freeze in place.

2.4 Setting the Stressmeter

The stressmeter is shipped with two snap-ring anchors mounted on the back end. These anchors are retained in their retracted position by means of a pull-pin. When the pull-pin is removed the snap-rings expand and grip the side of the borehole holding the stressmeter in place while the grout sets up. **Be careful not to trip the anchors prematurely.**

Mount the stressmeter on the setting tool by engaging the pin on the back of the stressmeter into the bayonet arrangement on the tool. The pin is aligned with the No. 1 wire. Connect a cable to the anchor pull-out pin and then connect the first setting rod to the setting head. Slowly, push the gage down the hole while maintaining orientation. Continue to add setting rods until the final location is reached. At this time, the orientations should be checked. Pin 1 is frequently aligned with the vertical or some other well-defined direction. It is desirable to take additional readings as a final check before setting. When the gage is ready to be set, pull on the anchor pull-pin cable using the setting rod for reaction. When the anchors have been set the pull-pin and cable are removed from the hole. Carefully disengage the setting tool and remove it and the rods from the borehole. Be sure to thoroughly clean the grout off the tool and rods. At this time, it is recommended to top-up the hole with grout to maintain some grout pressure in the gage area.

2.5 Taking Initial Readings

Readings should be taken at intervals immediately following installation to insure good zero data and also to see if the grout applies any small preload to the gage. The grout should gain strength according to the manufacturer's specifications and full strength should be achieved within a few days.

3. Taking Readings

3.1 Operation of the GK-403 Readout Box

The GK-403 Readout Box provides the necessary excitation and signal conditioning for the Model 4300 Series Stressmeters. To take readings, the box is connected to the gage by a jumper with either clip leads or, in the case of a terminal station, with a connector.

1. Turn the display selector to position "F" for EX size or position "B" for BX and NX sizes.

2. Turn the Readout Box on and a reading will appear in the front display window. The last digit may fluctuate by several digits and this is explained below.

3. Zeros in the display indicate a faulty connector, a damaged gage or high levels of electrical noise. Connect the ground lead to the cable shield (in this last case) and if the signal does not appear, trouble shooting is required (see Section 6).

4. The Readout Box will automatically turn off after approximately 4 minutes to conserve power.

As noted above, the last digit in the display will very often fluctuate by several digits, and this should not be seen as abnormal operation. In the case of the stressmeter, the vibrating wire is very short and the signals are not as pure as those of other gages. This, coupled with the fact that we are presenting frequency squared, causes some instability, which shows up in the least significant digit. This is not to say that the readings are not accurate; it simply means that the period of vibration changes very slightly from one pluck to the next. In most cases the displayed numbers should be rounded to the next least significant digit. For very stable stressmeters the last digit can give very valuable information on very small stress changes, and for that reason the numbers are not rounded off electronically.

3.2 Operation of the GK-404 Readout Box

The GK-404 is a palm sized readout box which displays the Vibrating wire value and the temperature in degrees Centigrade.

The GK-404 Vibrating Wire Readout arrives with a patch cord for connecting to the vibrating wire gages. One end will consist of a 5-pin plug for connecting to the respective socket on the bottom of the GK-404 enclosure. The other end will consist of 5 leads terminated with alligator clips. Note the colors of the alligator clips are red, black, green, white and blue. The colors represent the positive vibrating wire gage lead (red), negative vibrating wire gage lead (black), positive thermistor lead (green), negative thermistor lead (white) and transducer cable drain wire (blue). The clips should be connected to their respectively colored leads from the vibrating wire gage cable.

For **EX** size use the **POS** (Position) button to select position **F** and the MODE button to select **Dg** (digits).

For **BX and NX** sizes use the **POS** (Position) button to select position **B** and the **MODE** button to select **Dg** (digits).

Other functions can be selected as described in the GK404 Manual.

The GK-404 will continue to take measurements and display the readings until the OFF button is pushed, or if enabled, when the automatic Power-Off timer shuts the GK-404 off.
The GK-404 continuously monitors the status of the (2) 1.5V AA cells, and when their combined voltage drops to 2V, the message **Batteries Low** is displayed on the screen. A fresh set of 1.5V AA batteries should be installed at this point

4. Data Reduction

The GK-403 or GK 404 Readout Box excites the gage and times the period of 255 (or less) cycles of gage vibration using a 6.144 MHZ quartz oscillator, and displays the period to a resolution of 0.1 microseconds in Position "A". In Position "B" which is used for stressmeters, the processor converts the period readings to units of frequency squared, which is directly proportional to wire strain, gage deflection and applied stress.

The theory of the vibrating wire stressmeter is given in Appendix 2. The process of data reduction is as follows:

a. First, calculate the radial deformation V_1, V_2, and V_3 from equation 3. (Page 9)

b. From knowledge of the Young's Modulus and Poisson's Ratio for the rock, concrete or ice, refer to Figure 2 (or use calculations) to obtain values for the factors A and B.

c. Insert the values of V_1, V_2, V_3, A and B in equations 31, 32 and 33 to obtain p, q and θ.

d. Apply any correction required for longitudinal stress in the direction of the borehole axis. This is only possible with a stressmeter incorporating a longitudinal strain sensor measuring $\varepsilon \ell$.

Example

For a rock with a Young's Modulus of 5×10^6 psi and Poisson's Ratio of 0.3.

Initial readings on Channel B (Model GK-401 Readout Box) are:

Axis 1	6050
Axis 2	6235
Axis 3	6198

After application of stresses p and q readings are:

Axis 1	5020
Axis 2	6405
Axis 3	6400

From equation 3: $V_{r1} = 371 \times 10^{-6}$

$V_{r2} = -61 \times 10^{-6}$

$V_{r3} = -73 \times 10^{-6}$

From Figure 2 or by calculations: $A = 0.0325 \times 10^{-6}$

$B = 0.138 \times 10^{-6}$

From equations 31, 32 and 33: $p = 2273$ psi

$q = 157$ psi

$\theta = 0^{\circ}$

4.1 Gage Orientation

The No. 1 sensor coincides with the pin on the back end of the gage. No. 2 is 60º clockwise looking down the borehole and No. 3 is at 120º clockwise. (The Model 4350-2 has 6 radial sensors. Sensor 4 is lined up with sensor 1, sensor 5 is lined up with sensor 2, and sensor 6 is lined up with sensor 3. These extra sensors provide a measure of redundancy.)

4.2 Longitudinal Sensor

The Longitudinal Sensor measures the overall change in length of the stressmeter by using a vibrating wire strung from one end of the gage to the other. Assuming that the gage is fully bonded to the surrounding rock, the formula for longitudinal strain along the borehole is:

$$\varepsilon \ell = (R_0 = R_T), \text{ x } G \text{ microstrain}$$

Where, R_0 = initial reading (GK-401 or GK403 Position "B")

R_T = current reading

G = gage factor (supplied with the sensor)

For example: $R_O = 5{,}000$

$R_T = 4{,}800$

$\varepsilon \ell = (R_O - R_T) \text{ x } G$

$= (5{,}000 - 4{,}800) \text{ x } 0.3574$

$= 71.48$ micro-strain shortening of the borehole

The Longitudinal Sensor has a slight temperature sensitivity, which can be corrected for. Its thermal coefficient is 0.461 micro-strain/ºC decrease.

To correct for this, use the following:

$$\varepsilon \ell T = (R_0 - R_T) G + (T_T - T_0) \times 0.461$$

Note, as the temperature rises, the applied correction is positive.

To apply the correction for longitudinal strain, the increment $\nu_i \varepsilon_l E_i$ should be added to the values computed for p and q.

ν_i = Poisson's Ration of the rocks, concrete or ice.

E_i = Young's Modulus of the rocks, concrete or ice.

$\varepsilon \ell$ = longitudinal strain which, for a compressive strain, would be positive.

4.3 Temperature

Temperature corrections for the radial sensors are not normally required because their thermal sensitivity is practically zero in a free field, and corrections for changing temperatures in a restrained condition are very complex. Calibration in situ would be required for accurate determination of thermal characteristics.

4.4 Environmental Factors

Since the purpose of the stressmeter installation is to monitor site conditions, factors, which may affect these conditions, should always be observed and recorded. Seemingly minor effects may have a real influence on other behavior of the structure being monitored and may give an early indication of potential problems. Some of these factors include, but are not limited to: blasting, rainfall, tidal levels, excavation and fill levels and sequences, traffic, temperature and barometric changes, changes in personnel, nearby construction activities, seasonal changes, etc.

5. Troubleshooting

Maintenance and troubleshooting of vibrating wire stressmeters is confined to periodic checks of cable connections and maintenance of terminals. The transducers themselves are sealed and cannot be opened for inspection.

If a unit fails to read, the following steps should be taken:

1. Check the coil resistance. Nominal coil resistance is 90 Ω (180 for temp. and longitudinal sensors) ±5 Ω, plus cable resistance (22 gage copper = approximately 20 Ω per 1000 feet).
    a. If the resistance is high or infinite, a cut cable must be suspected.
    b. If the resistance is low or near zero, a short must be suspected.
    c. If resistances are within the nominal range and no reading is obtained, the transducer is suspect and the factory should be consulted.
    d. If all resistances are within nominal range and no readings are obtainable on <u>any</u> transducer, the readout is suspect and the factory should be consulted.
2. If cuts or shorts are located, the cable may be spliced in accordance with recommended procedures.

Appendix.1

Specifications

Specifications

Range (compression only)	30,000 psi
Approximate Sensitivity* (Hydrostatic)	5.4 psi
Accuracy (Hydrostatic)	± 0.1% F.S.
Operating Temperature Standard	–30 to +65ºC
High Temperature Option	–60º to +200ºC
Frequency Range	1400-3500 Hz
Borehole Diameter	BX, (2.36 inch), (60mm)
Dimensions	
R_2	1.125 in. (28.6mm)
R_1	0.650 in. (16.5mm)
R_c	1.000 in. (25.4mm)
Length with end caps	12.5 in. (317.5mm)
Length without end caps	7.0 in. (177.8mm)
Weight	7 lbs. (15.4kgm)
Optional Sensors	
VW Longitudinal Deformation Sensor:	
Range	2,000 microstrain.
Sensitivity	0.4 microstrain.
VW Temperature Sensors:	
Range	–60º to +200ºC
Sensitivity	0.035ºC
Accessories	
Setting Rods – 6 feet or 2 meter, self-orienting	
Setting Head	
Anchor Actuating Cable	

*With GK-401 Readout Box on Channel B

Appendix 2.

Biaxial Stress Theory

Biaxial Stress Theory

The measurement of stresses in rock, concrete or ice with an embedded sensor requires precise knowledge of the strain in the sensor, the stress-strain relationship of the sensor material and the sensor's inclusion factor under different loading conditions. Fortunately, we know the modulus of the biaxial steel sensor and we can precisely determine the gage deformation using vibrating wire technology. Analytical solutions are also available that describe the behavior of a cylindrical inclusion in a plate under loading.

Since we are generally interested in compressive stress, compressive displacements and stresses are taken to be positive as is often done in rock mechanics. Principal stresses are designated by p and q. The major principal stress, p, is the larger compressive stress, such that $p > q$. All angles are measured clockwise from the p direction.

Gage Deformation

The diametral deformation of the gage is determined by measuring the resonant frequency of each of the three vibrating wires. The fundamental frequency of each wire is proportional to the strain in the wire and is related to the wire strain by the equation:

$$f = \frac{1}{2\ell_w}\sqrt{\frac{\varepsilon E_w}{p_w} g} \qquad \text{(Equation 1)}$$

where, f = natural frequency of the wire (s^{-1})

ℓ_w = wire length (5.08×10^{-2} m) (2.00 ins.)

ε = wire strain

E_w = wire modulus (207 Gpa) (30×10^{6} psi)

p_w = wire density (7.83×10^{3} kg/m^{3}) (0.283 lbs./cu.ins)

g = acceleration of gravity (386 in./sec./sec.)

Equation 1 may also be expressed as:

$$E = kf^2$$

or

$$\Delta\varepsilon = k\,\Delta(f^2)$$

where

$$G = \frac{4\ell^2_w p_w}{E_w g} \quad \text{(theoretical)} \qquad \text{(Equation 2)}$$

$$\Delta(f^2) = (f_0^2 - f_T^2) = (R_0 - R_T) \times 10^3 / \sec^2$$

where R_0 and R_T are the readings on Channel B on the GK-401 Readout Box at time zero and time T.

In practice, stressmeters are calibrated at the factory using a hydraulic test chamber to exert known radial pressures and measure actual wire deformations. In this way, an empirically derived G factor is measured and used. For the standard BX size stressmeter, the factor G is approximately 0.36×10^{-9} $\sec.^2$. The actual calibration factor G is supplied with the gages.

Since the radial deformation of the cylinder,

$$V_r \text{ at } \ell_w/2 \text{ radius is equal to } V_r = (\ell_w/2)\,\Delta\varepsilon$$

we have:

$$V_r = (\ell_w/2)\, G\, \Delta\,(f^2)$$

and since,

$$\ell_w = 2.00 \text{ ins.}$$

$$\Delta\,(f^2) = (R_0 - R_T) \times 10^3 / \sec^2$$

$$V_r = G\,(R_0 - R_T) \times 10^{-6} \text{ ins.} \qquad \text{(Equation 3)}$$

Stresses Associated with Cylindrical Sensors

The stress-deformation relationship for cylindrical elastic inclusions in elastic and viscoelastic materials has been examined both analytically and experimentally. Savin (1961), Berry and Fairhurst (1966), Williams (1973) and others have developed analytical solutions for elastic materials. Experimental tests have verified that the analytical solutions accurately describe the stress distribution in an elastic plate (Suzuki 1969, Wilson 1961). The analytical solutions also describe the deformation of cylindrical elastic inclusions in viscoelastic and other time-dependent materials in uniaxial and biaxial loading experiments (Hawkes 1969a, b, Skilton 1971, Williams 1973, Busell et al. 1975, Johnson and Cox 1980).

The stress and displacement equations used in the following to describe the behavior of the biaxial stress sensor and surrounding rock, concrete or ice are based on the work of Savin (1961), who developed a set of analytical equations to describe the behavior of an elastic ring welded in an elastic plate. Even though the rock, concrete or ice has time-dependent properties, the analytical results of Berry and Fairhurst and the experimental work of Hawkes, Skilton, and Buswell indicate that Savin's equations can still be used.

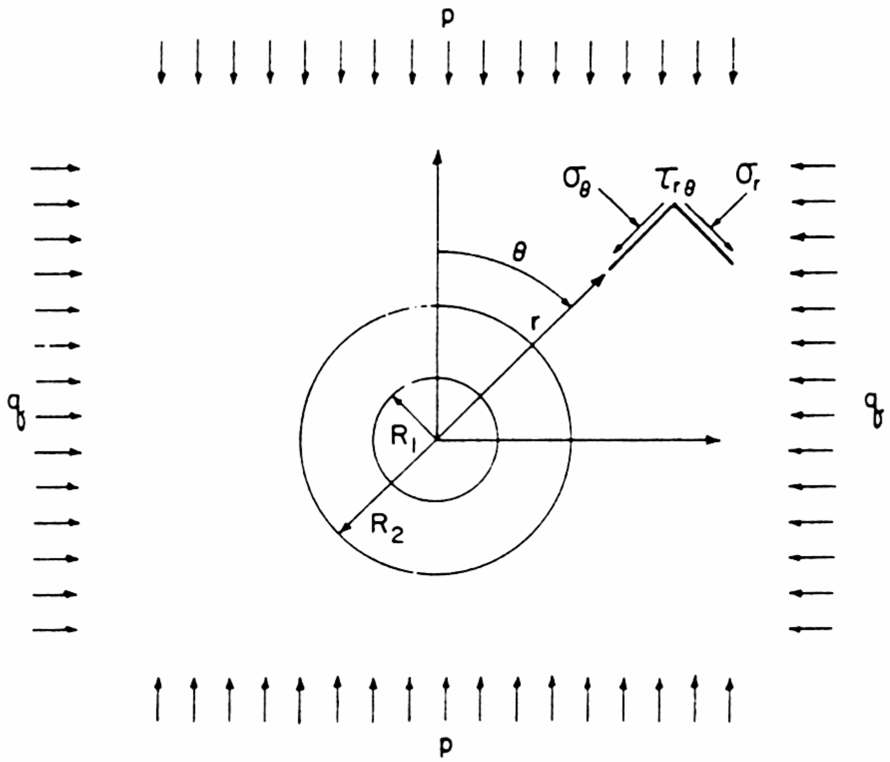

Figure 1. Plan view of cylindrical sensor embedded in rock, concrete or ice.

Generally, we are interested in measuring in-plane stresses in the rock, concrete or ice. Consider a cylindrical sensor that is embedded into an infinite isotropic medium (Fig. 1). The sensor is oriented normal to the plane, which is subjected to in-plane principal stresses p and q. The sensor has an outer radius R_2 and an inner radius R_1.

The stress (σ_r, σ_θ and $\tau_{r\theta}$) and displacement (V_r and V_θ) equations for the sensor ($R_1 < r < R_2$) in polar coordinates are:

$$\sigma_r = \left(\frac{p+q}{2}\right)\left(C_2 - \frac{C_5}{2}\,\frac{R_2^2}{r^2}\right) + \left(\frac{p-q}{2}\right)\left(\frac{C_7}{2} - 2C_1\,\frac{R_2^2}{r^2} - \frac{3}{2}\,C_4\,\frac{R_2^4}{r^4}\right)\cos 2\theta \tag{4}$$

$$\sigma_\theta = \left(\frac{p+q}{2}\right)\left(C_2 + \frac{C_5}{2}\,\frac{R_2^2}{r^2}\right) - \left(\frac{p-q}{2}\right)\left(\frac{C_7}{2} - 6C_3\,\frac{r_2^2}{R_2} - \frac{3}{2}\,C_4\,\frac{R_2^4}{r^4}\right)\cos 2\theta \tag{5}$$

$$\tau_{r\theta} = \left(\frac{p-q}{2}\right)\left(3C_3\,\frac{r^2}{R_2^2} - \frac{C_7}{2} - C_1\,\frac{R_2^2}{r^2} - \frac{3}{2}\,C_4\,\frac{R_2^4}{r^4}\right)\sin 2\theta \tag{6}$$

$$V_r = \frac{(p+q)}{8\mu_s}\,R_2\left(C_2\,(X_s - 1)\,\frac{r}{R_2} + C_5\,\frac{R_2}{r}\right) + \frac{(p-q)}{8\mu_s}\,R_2\left(C_3\,(X_s - 3)\,\frac{r^3}{R_2^3} + C_7\,\frac{r}{R_2}\right.$$
$$\left. + \,C_1\,(X_s + 1)\,\frac{R_2}{r} + C_4\,\frac{R_2^3}{r^3}\right)\cos 2\theta \tag{7}$$

$$V_\theta = \frac{(p-q)}{8\mu_s}\,R_2\left(C_3\,(X_s + 3)\,\frac{r^3}{R_2^3} - C_7\,\frac{r}{R_2} - C_1\,(X_s - 1)\,\frac{R_2}{r} + C_4\,\frac{R_2^3}{r^3}\right)\sin 2\theta \tag{8}$$

The stress and displacement equations for the surrounding medium (r > R_2) are:

$$\sigma_r = \left(\frac{p+q}{2}\right)\left(1 - \frac{C_6}{2}\,\frac{R_2^2}{r^2}\right) + \left(\frac{p-q}{2}\right)\left(1 - 2C_8\,\frac{R_2^2}{r^2} - \frac{3}{2}\,C_9\,\frac{R_2^4}{r^4}\right)\cos 2\theta \qquad (9)$$

$$\sigma_\theta = \left(\frac{p+q}{2}\right)\left(1 + \frac{C_6}{2}\,\frac{R_2^2}{r^2}\right) - \left(\frac{p-q}{2}\right)\left(1 - \frac{3}{2}\,C_9\,\frac{R_2^4}{r^4}\right)\cos 2\theta \qquad (10)$$

$$\tau_{r\theta} = -\left(\frac{p-q}{2}\right)\left(1 + C_8\,\frac{R_2^2}{r^2} + \frac{3}{2}\,C_9\,\frac{R_2^4}{r^4}\right)\sin 2\theta \qquad (11)$$

$$V_r = \frac{(p+q)}{8\mu_i}\,R_2\left((X_i - 1)\,\frac{r}{R_2} + C_6\,\frac{R_2}{r}\right)$$

$$+ \frac{(p-q)}{8\mu_i}\,R_2\left(2\,\frac{r}{R_2} + C_8\,(X_i + 1)\,\frac{R_2}{r} + C_9\,\frac{R_2^3}{r^3}\right)\cos 2\theta \qquad (12)$$

$$V_\theta = \frac{(p-q)}{8\mu_i}\,R_2\left(-2\,\frac{r}{R_2} - C_8\,(X_i - 1)\,\frac{R_2}{r} + C_9\,\frac{R_2^3}{r^3}\right)\sin 2\theta. \qquad (13)$$

The coefficients C_1 through C_9 depend on the sensor geometry and the material properties of the sensor and rock, concrete or ice where:

$$C_1 = 2\left(\frac{1+X_i}{D}\right)\left[\left(\frac{\mu_i}{\mu_s} - 1\right) + n^6\left(1 + X_s\,\frac{\mu_i}{\mu_s}\right)\right] \qquad (14)$$

$$C_2 = \frac{n^2(1+X_1)}{2\left(\frac{\mu_i}{\mu_s} - 1\right) - n^2\left[\left(\frac{\mu_i}{\mu_s} - 1\right) - \left(1 + X_s\,\frac{\mu_i}{\mu_s}\right)\right]} \qquad (15)$$

$$C_3 = -2\,\frac{(1+X_i)}{D}\,n^4\,(n^2-1)\left(\frac{\mu_i}{\mu_s} - 1\right) \qquad (16)$$

$$C_4 = -2\,\frac{(1+X_1)}{D}\left[\left(\frac{\mu_i}{\mu_s} - 1\right) + n^4\left(1 + X_s\,\frac{\mu_i}{\mu_s}\right)\right] \qquad (17)$$

$$C_5 = \frac{2C_2}{n^2} \qquad (18)$$

$$C_6 = 2 - 2\,\frac{(n^2-1)}{n^2}\,C_2 \qquad (19)$$

$$C_7 = 2\,\frac{(1+X_i)}{D}\left[\left(\frac{\mu_i}{\mu_s} - 1\right)(4 - 3n^2) + n^6\left(1 + X_s\,\frac{\mu_i}{\mu_s}\right)\right]n^2 \qquad (20)$$

$$C_8 = 2-2\,\frac{(1+X_i)}{D}\left[\left(\frac{\mu_i}{\mu_s}-1\right)(3n^6-6n^4+4n^2-1)+n^6(n^2-1)\left(1+X_s\frac{\mu_i}{\mu_s}\right)\right] \qquad (21)$$

$$C_9 = -2+2\,\frac{(1+X_i)}{D}\left[\left(\frac{\mu_i}{\mu_s}-1\right)(4n^6-7n^4+4n^2-1)+n^4(n^4-1)\left(1+X_s\frac{\mu_i}{\mu_s}\right)\right] \qquad (22)$$

In the above equations

$$n = \frac{R_2}{R_1}$$

$$\mu = \frac{E}{2(1+v)}$$

$$X = \frac{(3-v)}{(1+v)} \qquad \text{for plane stress}$$

$$X = (3-4v) \qquad \text{for plane strain}$$

$$D = \left(X_i+\frac{\mu_i}{\mu_s}\right) n^2 \left[\left(\frac{\mu_i}{\mu_s}-1\right)(3n^4-6n^2+4)+n^6\left(1+X_s\frac{\mu_i}{\mu_s}\right)\right]$$
$$+\left(X_s\frac{\mu_i}{\mu_s}-X_i\right)\left[\left(\frac{\mu_i}{\mu_s}-1\right)+n^6\left(1+X_s\frac{\mu_i}{\mu_s}\right)\right] \qquad (23)$$

E is Young's modulus and ν is Poisson's ratio where the subscripts s and i denote the material properties of the sensor and surrounding medium respectively. θ is the angle measured clockwise from the principal stress direction p to the direction of Sensor No.1.

Determination of Stresses in the Medium

The magnitude and direction of the principal stresses in the medium are determined from the measured radial deformation of the sensor in three directions. In the biaxial stress sensor the measurement directions are 120° apart and from eq 7 we have for the displacements of the three wires:

$$V_{r1} = A(p+q) + B(p-q)\cos 2\theta_1 \qquad (24)$$

$$V_{r2} = A(p+q) + B(p-q)\cos 2\theta_2 \qquad (25)$$

$$V_{r3} = A(p+q) + B(p-q)\cos 2\theta_3 \qquad (26)$$

where

$$A = \frac{R_2}{8\mu_s}\left[C_2(X_s-1)\frac{R_c}{R_2}+C_5\frac{R_2}{R_c}\right] \qquad (27)$$

$$B = \frac{R_2}{8\mu_s}\left[C_3(X_s-3)\frac{R_c^{\,3}}{R_2^{\,3}}+C_7\frac{R_c}{R_2}+C_1(X_s+1)\frac{R_2}{R_c}+C_4\frac{R_2^{\,3}}{R_c^{\,3}}\right] \qquad (28)$$

$$\theta_2 = \theta_1 + 60^{\circ} \qquad (29)$$

and

$$\theta_3 = \theta_1 + 120^{\circ} \qquad (30)$$

In the above equations, θ_1 is the angle measured clockwise from principal stress direction p to the measurement direction V_{r1} ; $2R_c$ is the length of the vibrating wire sensor, (1inch).

Solving for p, q and θ, we obtain

$$p = \frac{1}{2}\left[\frac{1}{3B}\left((2V_{r_1}-V_{r_2}-V_{r_3})^2+3(V_{r_2}-V_{r_3})^2\right)^{1/2}+\frac{1}{3A}(V_{r_1}+V_{r_2}+V_{r_3})\right] \qquad (31)$$

$$q = \left[\frac{1}{3A}(V_{r_1}+V_{r_2}+V_{r_3})-p\right] \qquad (32)$$

and

$$\theta_1 = \frac{1}{2}\cos^{-1}\left[\frac{V_{r_1}-A(p+q)}{B(p-q)}\right]$$

But, because

$$\cos(\theta) = \cos(-\theta).$$

Equation 33 has two solutions.

If $V_{r2} = A(p+q) + B(p-q)\cos 2(\theta_1+60^{\circ})$, then θ_1 is positive.

But. if $V_{r2} = A(p+q) + B(p-q)\cos 2(\theta_1+120^{\circ})$ then θ_1 is negative.

In general the approximate location of the p direction can be intuited by examination of the relative magnitude of the measured diametral changes so that any ambiguity about the sign of θ can be resolved in this way.

The coefficients A and B depend upon the geometry and mechanical properties of the sensor and the mechanical properties of the surrounding medium. A and B for the biaxial stress sensor are plotted again Young's modulus of the surrounding medium for different Poisson's ratios in Figure 2

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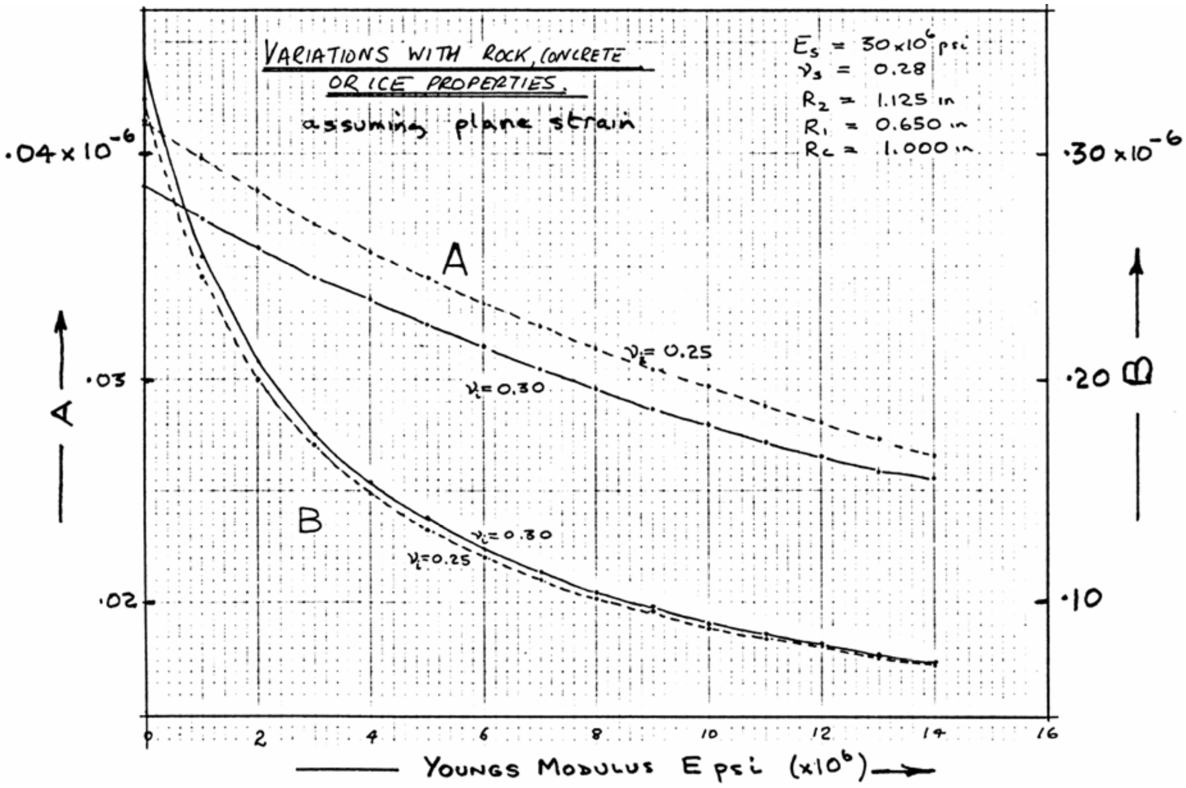

Figure 2.

A and B have the units inch^3 / lbf

To convert to metric, multiply A and B by 3684 x mm^3 /Newton. (Or 3.684 x 10^{-6} meters^3 /Newton)

To convert to metric , multiply E psi by 0.006895 MPa

To convert from metric multiply mm^3/Newton by 0.0002714 to get inch^3 / lbf

(Or from meters^3 /Newton multiply by 2.714 x 10^5 to get inch^3 / lbf

To convert from metric multiply GPa by 0.1451 to get psi x 10^6

Appendix 3.

Wiring Code for 6 Gage Biaxial Stress Meter

VW Biaxial Stressmeter: Model 4350-3

The Stressmeter uses a 6-pair orange-colored cable and each pair has a black lead. Careful attention to the wire code is a must to obtain correct readings.

Sensor Number (Looking Down Hole)			
1	0º Radial 1	Red	Pair
2	60º Radial 2	Red's/Black	
3	120º radial 3	Brown	Pair
L_1	Longitudinal 1	Brown's Black	
L_2	Longitudinal 2	Yellow	Pair
T_1	Temperature 1	Yellow's Black	
4	0º Radial 4	Blue	Pair
5	60º Radial 5	Blue's Black	
6	120º Radial 6	Green	Pair
T_2	Temperature 2	Green's Black	
	Common lead for 1, 2, 3, L_1, L_2, T_1 (used with 3 radial gages)	White's Black	Pair
	Common lead for 4,5,6,T_2, (used with 6 radial gages)	White	

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Wiring Code for 3 Gage Biaxial Stress Meter

VW Biaxial Stressmeter: Model 4350-1

The Stressmeter uses a 4-pair violet-colored cable and each pair has a black lead. Careful attention to the wire code is a must to obtain correct readings.

Sensor Number (Looking Down Hole)			
1	0º Radial 1	Red	Pair
1	0º Radial 1	Red's/Black	
2	60º radial 2	White	Pair
2	60º radial 2	White's Black	
3	120º radial 3	Green	Pair
3	120º radial 3	Green's Black	
T	Thermistor	Blue	Pair
T	Thermistor	Blue's Black	
S	Shield	Shield	All

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Wiring Code for 3 Gage Biaxial Ice Gage

VW Biaxial Ice Gage: Model 4350-X

The Ice Gage uses a 4-pair TPR cable and each pair has a black lead. Careful attention to the wire code is a must to obtain correct readings.

Sensor Number (Looking Down Hole)			
1	0º Radial 1	Brown	Pair
1	0º Radial 1	Brown's/Black	
2	60º radial 2	Red	Pair
2	60º radial 2	Red's Black	
3	120º radial 3	Green	Pair
3	120º radial 3	Green's Black	
T	Thermistor	Yellow	Pair
T	Thermistor	Yellow's Black	
S	Shield	Shield	All

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