Model 3200

Borehole Pressure Cell

Instruction Manual

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1. INTRODUCTION

The Borehole Pressure Cell (BPC) is designed to monitor stress changes in rock materials, including hard rocks, coal, rock-salt, potash, etc. It consists of a flatjack made from two steel plates welded together at their edges with the intervening space filled with hydraulic oil. The cell has a "dog bone" cross section, allowing it to expand easily over a large range without splitting apart. A 6 mm (0.24") diameter tube connects directly to the BPC and on its outer end is T-bar used to orient the BPC perpendicular to the applied rock stress. A further length of 3 mm (0.12") diameter high-pressure steel tubing connects the tubing on the BPC to a pressure gauge and/or to a pressure transducer (see Figure 1). Cells are provided already filled with de-aired oil. De-airing is done because the cell is more responsive the more rigid it is. Setting rods are provided to push the BPC into the borehole and correctly orient it.

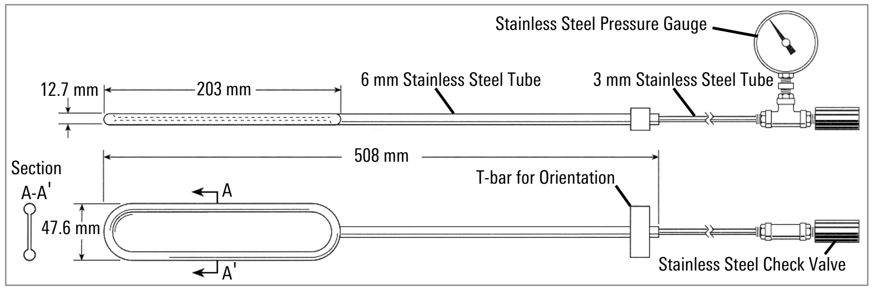

FIGURE 1: *Model 3200 Borehole Pressure Cell*

The figure below shows two different types of pressure measurement devices.

FIGURE 2: *Two Pressure Measurement Devices*

The figure below shows a pre-encapsulated BPC.

FIGURE 3: *Pre-encapsulated BPC*

In use the cell is grouted into a borehole drilled into the rock and when the grout has hardened the cell is connected to a hydraulic pump and pressurized to a pressure level equal to the estimated in-situ stress level, plus 10%. (A check valve prevents the cell pressure from being lost when the hydraulic pump is disconnected.) Stress changes within the host material are transmitted to the cell causing a proportionate change in the internal hydraulic pressure, which is recorded on the pressure gage and/or pressure transducer, (vibrating wire or semi-conductor).

BPCs react mainly to stress changes in the direction perpendicular to the plain of the flatjack. They are relatively insensitive, (about 6%) to stress changes in the direction parallel to the plain of the flatjack. If stress changes are required in two directions, then two or three BPCs can be installed at different orientations in the same borehole or in adjacent boreholes.

2. INSTALLATION

2.1 BOREHOLE REQUIREMENTS

The Model 3200 BPC is designed for installation in 57 mm (2.25") diameter boreholes. Boreholes larger than this can be used but are not recommended. BPCs are normally installed at the back of the borehole.

2.2 GROUTING METHOD

Recommended grouts are:

- Portland cement
- Lumnite mortar
- Special Grout 400 (non-shrink)
- Sakrete (non-shrink)
- Quikrete (non-shrink)

Place grout in the borehole in one of two ways:

- Using downward-directed holes, tremie a neat cement (no sand) grout into the bottom of the borehole through a grout pipe.

  Note: Direct 'horizontal' boreholes downwards about three degrees

- Place grout in the back of the borehole using a series of rigid tubes/pipes which can be connected together. Fill the tube/pipe with mortar, which is pushed out of the pipe using a rod to act as a piston.

When the grout has been placed, push the cell into place on the end of the provided setting rods. The last section of the setting rods has a slot which engages the T-bar on the 6 mm (0.24") tube issuing from the BPC. The T-bar is used to orient the cell inside the borehole. When correctly oriented, the setting rods can be removed and the cell left in place until the grout sets.

2.3 ENCAPSULATION METHOD

Cells can be provided pre-encapsulated. Grouting is the preferred method of installation. If grouting is not possible, then the cells can be pre-encapsulated inside a cylinder of mortar. Make sure the cylinder is a close fit inside the 57 mm (2.25") diameter borehole. For this method the borehole should be smooth walled so that the encapsulation slides in easily. The encapsulated cell is expanded inside the borehole in the normal manner.

Push the cell into the borehole with the slot in the setting rods engaging the T-Bar on the cell. When correctly oriented, the setting rods can be left in place until the cell has been expanded.

2.4 PRESSURIZING THE BPC FLATJACK

The BPCs are supplied filled with de-aired hydraulic oil. Pressurize the cell using any manual hydraulic pump filled with oil. (GEOKON can provide a pump filled with de-aired oil, if necessary.)

The pump is attached to the BPC by removing the cap from the back end of the check valve. A special filler tube with mating fitting is connected to the check valve. While making this connection do not allow air to enter the system. Purge the filler tube completely of any air and allow both halves of the connection to fill with oil before screwing them together. This is important to keep the BPC as rigid as possible, so that it is more responsive to rock stress changes.

Inflate the BPC slowly, while observing the pressure on the pressure gauge, until the desired pressure is reached. (Estimated in situ stress plus 10%. The extra 10% is recommended because it is normal for the installed pressure to back off a little as the BPC beds in.) Grouted systems should reach this pressure in just a very few strokes of the pump, whereas, encapsulated systems may take several strokes especially if the hole is oversize.

When the desired pressure is reached, disconnect the pump and replace the cap on the check valve. Use Teflon tape on the threads and tighten firmly.

3. DATA MANAGEMENT

Conversion of the measured BPC pressure changes to the equivalent rock stress change depends on the stiffness of the BPC, the modulus of the host rock and the degree of plasticity of the host rock. In general, the best results are obtained using the highest initial BPC installation pressure. Most practitioners use an initial pressure of 110% of the estimated in-situ rock stress. 110% is chosen because it is normal for the BPC to lose a little pressure as the BPC and the grout 'beds in', i.e., removes high local shearing stresses by deforming until equilibrium is restored. Using high initial pressures is, in effect, making the BPC as stiff and as responsive as possible. GEOKON further enhances this stiffness by using de-aired oil inside the BPC.

The paper reproduced in Appendix B shows the results of some tests and indicate the variability that can be expected. It also suggests a method by which the BPC response might be calculated, but the solution requires a number to represent the stiffness of the BPC and its derivation requires a special screw pump that is no longer used because the procedure is too tedious.

A compilation of tests results from various elastic rocks with varying modus, E, lead to the following relationship: The BPC response R% (The ratio of the BPC pressure change to the corresponding rock stress change), is given by the formula $R\% = -5.6E + 84$. Thus, for a rock with a modulus of 5×10^6 psi, $R\% = -(5 \times 5.6) + 84 = 56\%$, i.e., a rock stress change of 100 psi would result in a BPC pressure change of 56 psi. This equation is from tests using relatively low initial pressures and is given merely as matter of interest. Where the recommended higher initial pressures are used then the expected BPC response will be higher, approaching the 80% to 90% level.

In plastic rocks, such as evaporates, (salt, trona, potash), there is reason to believe that BPC response will be even higher, (in the 120% to 160% level) and studies in coal, which has fractured under high pressures, show responses of higher than 100%. It might be that the pressure inside a BPC, left in plastic rock, could eventually reach the in-situ stress.

4. TROUBLESHOOTING

The pressure gauge should be positioned in a place where it can be easily read. Wedge the gauge and Swagelok Tee in place so that they cannot be easily disturbed.

Cell pressures usually tend to bleed-off over the first few days as the cell beds in; this could be compensated for, by over-pressuring the cell during the initial pump up. If the loss of pressure is excessive then the cell may require re-pressurizing. If the cell loses all of its pressure, then this is indicative of a leak. Check all the visible connections for signs of leaking oil; tighten any connections that appear to be leaking. If none are visible then perhaps the flatjack is leaking, in which case the cell must be abandoned.

APPENDIX A. SPECIFICATIONS

A.1 3200 SPECIFICATIONS

Standard Ranges	5, 7.5, 10, 20, 35, 75, 100 MPa
Resolution	0.25% of range (approximately)
Accuracy[1]	0.25% F.S. (gauge); 0.1% F.S. (VW transducer)
Borehole Size	57 mm (2.25")
Temperature Range[2]	−20 to +80 °C
L x W x H	203 × 47.6 × 12.7 mm (8 × 1.875 × 0.5") (borehole flatjack only) 508 × 47.6 × 12.7 mm (20 × 1.875 × 0.5") (borehole flatjack with stainless steel tube)

TABLE 1: *Specifications*

Notes:

[1] VW transducer accuracy established under laboratory conditions

[2] Other ranges available on request

A.2 3200 PARTS LIST

3200WIKA–5mpa	Hydraulic Borehole Pressure Cell w/WIKA gauge	5 MPa
3200WIKA–7.5mpa	Hydraulic Borehole Pressure Cell w/WIKA gauge	7.5 MPa
3200WIKA–10mpa	Hydraulic Borehole Pressure Cell w/WIKA gauge	10 MPa
3200WIKA–20mpa	Hydraulic Borehole Pressure Cell w/WIKA gauge	20 MPa
3200WIKA–35mpa	Hydraulic Borehole Pressure Cell w/WIKA gauge	35 MPa
3200WIKA–75mpa	Hydraulic Borehole Pressure Cell w/WIKA gauge	75 MPa
3200WIKA–100mpa	Hydraulic Borehole Pressure Cell w/WIKA gauge	100 MPa
3200HH–5mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	5 MPa
3200HH–7.5mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	7.5 MPa
3200HH–10mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	10 MPa
3200HH–20mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	20 MPa
3200HH–35mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	35 MPa
3200HH–75mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	75 MPa
3200HH–100mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	100 MPa
3200B–5mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	With WIKA gauge, 5 MPa
3200B–7.5mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	With WIKA gauge, 7.5 MPa
3200B–10mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	With WIKA gauge, 10 MPa
3200B–20mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	With WIKA gauge, 20 MPa
3200B–35mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	With WIKA gauge, 35 MPa
3200B–75mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	With WIKA gauge, 75 MPa
3200B–100mpa	Hydraulic Borehole Pressure Cell for 4500HH Sensor	With WIKA gauge, 100 MPa

TABLE 2: *3200 Parts List*

Int. J. Rock Mech. Min. Sci. Vol. 7, pp. 423–435. Pergamon Press 1970. Printed in Great Britain

THE MEASUREMENT OF ROCK STRESS CHANGES USING HYDRAULIC BOREHOLE GAUGES

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Abstract—Hydraulic borehole instruments can be used for the measurement of rock stress changes. The following paper describes how the observed fluid pressure changes on the instrument gages can be converted to actual rock stress changes. Laboratory investigations were performed on eight different rock types using two different types of hydraulic borehole instruments. Conclusive evidence is presented to show that the conversion of fluid pressure changes to rock stress changes can be made with confidence as soon as the elastic constants of the rock and the stiffness of the hydraulic borehole instrument are known.

The procedure for determining the stiffness of a hydraulic borehole instrument is described and the theoretical formulae required to permit fluid pressure/rock stress conversions are developed and presented.

INTRODUCTION

THERE are several types of hydraulic gages which, when pressurized inside a borehole drilled in rock, can be used to detect rock stress changes. For example, there are the USBM cylindrical pressure cell designed to measure the *in situ* modulus of rigidity [1]; the USBM Borehole Pressure Cell (flatjack type) [2]; the Menard Pressuremeter for measuring the deformability of soils and soft rocks [3]; the NCB gage [4]; and many others. All such gages react to changing rock pressures but generally, the pressure change of the hydraulic fluid is not equal to the rock stress change. The purpose of this paper is to show how the rock stress change can be calculated from the observed hydraulic pressure change.

THEORY

Stress changes in the rock around a borehole give rise to pressure changes in the hydraulic cell within the borehole. Consider first the case of the hydraulic cell which exerts a uniform radial pressure on the walls of the borehole. By superposition, the stress distribution in the rock around the borehole is:

$$\sigma_r = \frac{S+T}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{S-T}{2}\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta+\frac{a^2}{r^2}P \tag{1}$$

$$\sigma_\theta = \frac{S+T}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{S-T}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta-\frac{a^2}{r^2}P. \tag{2}$$

For S, T and P, we can substitute ΔS, ΔT which are the rock stress changes at a distance from the borehole and ΔP the change in hydraulic pressure. a is the radius of the borehole and r and θ are the polar coordinates describing the position at which the radial stress σ_r and the tangential stress σ_θ are measured.

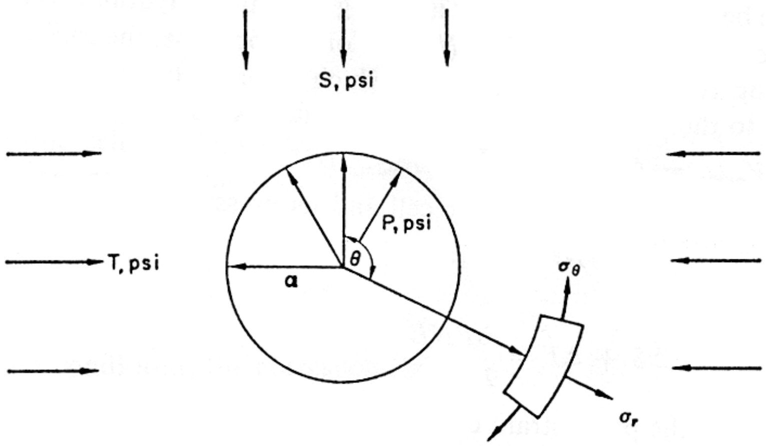

FIG. 1. Stresses around a hydraulic borehole cell which exerts a uniform radial pressure on the borehole wall.

Following the usual analysis in which stresses are converted to strains using Hooke's Law, and then integrating to obtain displacements, it can easily be shown that the displacement of the walls of the borehole, under conditions of plane stress, is given by

$$\Delta u_{r-a} = \frac{a}{E}[\Delta S + \Delta T + 2(\Delta S - \Delta T)\cos 2\theta - \Delta P(1 + \nu)] \tag{3}$$

where E and ν are the elastic constants of the rock. This can be converted into a change of borehole volume observing that the volume change is given by

$$\int_0^{2\pi} a\Delta u \mathrm{d}\theta$$

neglecting small quantities and is equal to

$$\Delta V = \frac{2\pi La^2}{E}[\Delta S + \Delta T - \Delta P(1 + \nu)] \tag{4}$$

where L is the length of the hydraulic gage.

This change in volume can be equated to changes in volume of the hydraulic fluid ΔV_f, and to changes in the volume of the confines of the hydraulic fluid ΔV_{c_1}, i.e.

$$\Delta V = \Delta V_{f_1} + \Delta V_{c_1} \tag{5}$$

For instance, if the borehole contracts, then the hydraulic fluid volume also contracts whereas the tube connecting the borehole cell to the pressure measuring gage and the pressure gage itself expands.

To calibrate the gage, we must artificially alter the volume of the hydraulic cell and measure the pressure change this causes. To do this, we can use a positive displacement screw pump connected to the hydraulic cell while the cell is in place in the borehole. Under these conditions, the total volume change of the system is given by

$$\Delta V_t = (\Delta V_{f_1} + \Delta V_{f_2}) + (\Delta V_{c_1} + \Delta V_{c_2}) + \frac{2\pi La^2}{E}(\Delta P(1 + \nu)) = N_t\Delta P \tag{6}$$

where N_t is the measured change in volume per unit pressure change with the pump connected in the system, and ΔV_{c_2} and ΔV_{f_2} are the changes of volume in fluid confines and hydraulic fluid of the pump part of the system only.

The effect of the pump can be measured directly by isolating it from the hydraulic cell and again measuring the pressure/volume characteristic, N_{pump}. For this purpose, the calibration procedure is conducted using two pressure gages, one which remains connected to the hydraulic cell and the other to the pump. With the pump and its pressure gage alone

$$\Delta V_{\text{pump}} = \Delta V_{f_2} + \Delta V_{c_2} = N_{\text{pump}}\,\Delta P. \tag{7}$$

The pressure/volume relationship of the hydraulic cell and its pressure gage is given by

$$N = N_t - N_{\text{pump}}. \tag{8}$$

Combining equations (4)–(8) gives

$$\Delta S + \Delta T = \frac{\Delta PNE}{2\pi La^2}. \tag{9}$$

A similar line of argument for the plane strain condition leads to the relationship

$$\Delta S + \Delta T = \frac{\Delta PNE}{2\pi La^2\,(1 - \nu^2)}. \tag{10}$$

So, if N, E and ν can be measured, then $\Delta S + \Delta T$ can be calculated from observed fluid pressure changes ΔP. Note that a gage which exerts a uniform radial pressure on the walls of the borehole cannot distinguish between ΔS and ΔT. However, if the gage is situated in a pillar, one might infer that ΔT, the horizontal stress is zero.

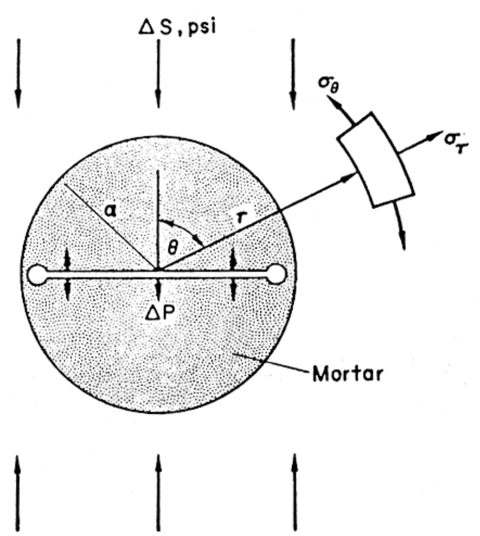

FIG. 2. Stresses on the borehole flatjack type of hydraulic cell.

The second type of gage considered here is the borehole flatjack type. Following the same line of argument as before, we can equate the stress change distribution in the rock around the borehole as follows:

$$\sigma_r = \frac{\Delta S}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\Delta S}{2}\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos 2\theta + \frac{a^2}{r^2}\,\Delta P\cos^2\theta \tag{11}$$

$$\sigma_\theta = \frac{\Delta S}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\Delta S}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta - \frac{a^2}{r^2}\,\Delta P\sin^2\theta. \tag{12}$$

Assuming plane stress conditions, converting stresses to strains via Hooke's Law, integrating to obtain displacements and integrating to obtain the volume change of the borehole, we obtain

$$\Delta V = \frac{\pi L a^2}{E} [2\Delta S - \Delta P(1 + \nu)]. \tag{13}$$

Again the hydraulic cell can be calibrated by finding its pressure/volume characteristics using a screw pump, and the equation for obtaining the rock stress change from the observed fluid pressure change is:

$$\Delta S = \frac{\Delta PNE}{2\pi L a^2}. \tag{14}$$

For plane strain conditions, the corresponding relationship is:

$$\Delta S = \frac{\Delta PNE}{2\pi L a^2 (1 - \nu^2)}. \tag{15}$$

The response of the flatjack type gage to rock stress changes in a direction parallel to the plane of the flatjack cannot be calculated in the same manner as before because resistance to closure of the hole is provided primarily by unknown stresses set up in the grouting medium. Fluid pressure changes in the flatjack are a result of the expansion or contraction of the grout in the direction perpendicular to the direction of the applied rock stress changes. Moreover, some of this expansion and contraction is accommodated by similar movements of the borehole walls in the same direction.

The problem might be amenable to solution using the theory of stress distribution in and around a solid circular inclusion present in a uniformly stressed body. The success of the analysis will depend on how well the complex grout–flatjack system can be represented by a homogeneous isotropic elastic inclusion. The formulae for stresses in a solid circular inclusion are given in the Appendix for both plane stress and plane strain situations. The formulae as presented here are in a simpler form than usually encountered and ought to be of general interest to the reader.

EXPERIMENTAL RESULTS

An extensive series of laboratory tests were conducted to measure the actual responses to rock stress changes of two types of hydraulic gages in several different kinds of rock. The measured responses were compared with responses calculated using the theory just presented. The two types of gage were the Cylindrical Pressure Cell (CPC), developed by the USBM for the measurement of the *in situ* modulus of rigidity of rock and the Borehole Pressure Cell (BPC), a type of borehole flatjack also developed by the USBM.

The cylindrical pressure cell is designed for a 1½ in. diameter borehole. The pressure is exerted in a uniform radial manner by means of a thin copper jacket which confines the fluid and which can be expanded against the walls of the borehole. The borehole pressure cell is a steel flatjack designed for grouting within a 2¼ in. diameter borehole. Both CPC and BPC have an active length of 7 in.

The tests were run using rock cores 6 in. in diameter and 10 in. long, with an axial hole 1½ in. in diameter. These cores were installed within the triaxial testing apparatus described by Obert [5]. This apparatus permitted pressures to be exerted on the rock cylinders both axially and radially [6]. The first series of tests was one in which strain gages cemented around the periphery of the cores and a borehole deformation gage [7] located in the axial $1\frac{1}{2}$ in. diameter borehole, were used to measure the elastic moduli of the rock cores. Table 1 gives the elastic moduli of the various rock types used. Note that the moduli vary quite considerably, both with magnitude and direction of loading. In the tables 'axial loading' implies that the radial pressure applied to the rock was kept constant while the axial loads were varied; 'radial loading' implies the reverse. The values given are from the strain gage data. As a point of interest, the deviation of the two sets of elastic moduli as derived from the borehole deformation gage and the strain gages and expressed by the formulae $(E_{bdg} - E_{sg})/E_{sg}$ and $(\nu_{bdg} - \nu_{sg})/\nu_{sg}$ showed a mean of -1% with a standard deviation of $\pm 12 \cdot 5\%$.

After the elastic constants had been determined, they were redetermined using the cylindrical pressure cell and the method described by PANEK *et al.* [1]. Table 2 shows the Young's modulus as determined by the CPC. The CPC results are in good agreement with the previous results which in itself, is a verification of the accuracy of the CPC method of measuring the modulus of rigidity. Discrepancies that did occur were partly the result of the need for special techniques which only came to light during these tests, but a discussion of these techniques is beyond the scope of this paper.

The determination of elastic moduli using the CPC involved the cycling of CPC fluid pressures while maintaining constant radial and axial pressures on the rock core. Following these tests, the axial pressures were maintained constant while the radial rock pressures were varied and the corresponding CPC fluid pressures were observed. Table 3 shows the results summarized. The Table requires some explanation. L is simply the gage length in inches and a is the radius of the borehole in which the CPC was located. ν and E_{sg} are the elastic moduli of the rock calculated from strain gage data at the state of rock stress denoted

TABLE 1. YOUNG'S MODULUS AND POISSON'S RATIO FROM STRAIN GAGE MEASUREMENTS

Rock type	Young's modulus $\times$ 10^6psi				Poisson's ratio			
	Axial loading		Radial loading		Axial loading		Radial loading	
	$\sigma_z < 2000$	$\sigma_z > 2000$	$P_0 < 2000$	$P_0 > 2000$	$\sigma_z < 2000$	$\sigma_z > 2000$	$P_0 < 2000$	$P_0 > 2000$
Indiana limestone #1	5·24		4·98		0·26		0·26	
Indiana limestone #2	5·20		4·70		0·25		0·23	
Pennsylvania limestone #1	13·38		10·95		0·29		0·39	
Pennsylvania limestone #2	11·60		10·25		0·29		0·42	
Kansas salt #1	4·14		4·14		0·31		0·31	
Kansas salt #2	4·08		4·24		0·29		0·31	
Georgia granite #1	3·10		4·58		0·14		0·23	
Georgia granite #2	4·07	4·62	6·20	7·12	0·15	0·18	0·15	0·24
Ohio limestone #1	8·76	8·76	8·45	8·03	0·29	0·29	0·29	0·31
Ohio limestone #2	6·39	6·77	7·25	7·51	0·20	0·22	0·26	0·28
Pennsylvania diabase #1	15·75	15·75	15·77	15·77	0·19	0·19	0·30	0·30
Pennsylvania diabase #2	15·40	15·40	14·45	14·45	0·27	0·27	0·26	0·26
Ohio salt #1	4·13		4·25		0·22		0·22	
Ohio salt #2	3·67		3·84		0·21		0·24	
New Mexico potash #1	3·50		3·85		0·26		0·29	
New Mexico potash #2	3·41		3·80		0·26		0·29	

TABLE 2. YOUNG'S MODULUS FROM CYLINDRICAL PRESSURE CELL MEASUREMENTS

Rock type		From Table 1 Poisson's ratio	From Table 1 Young's modulus	CPC Young's modulus	$\left(\frac{E_{CPC}}{E_{sg}}\right) - 1 \times 100\%$
Indiana limestone	#1	0·26	4·98		
Indiana limestone	#2	0·23	4·70	5·41	+15
Pennsylvania limestone	#1	0·39	10·95	10·10	−8
Pennsylvania limestone	#2	0·42	10·25	9·2	−10
Kansas salt	#1	0·31	4·14	2·62	−37
Kansas salt	#2	0·31	4·24	4·08	−4
Georgia granite	#1	0·23	4·58	4·50	−2
Georgia granite	#2	0·24	7·12	7·09	−2
Ohio limestone	#1	0·31	8·03	7·64	−5
Ohio limestone	#2	0·28	7·38	8·20	+11
Pennsylvania diabase	#1	0·30	15·77	14·10	−10
Pennsylvania diabase	#2	0·26	14·45	13·20	−9
Ohio salt	#1	0·22	4·25	3·38	−20
Ohio salt	#2	0·24	3·86	4·27	+12
New Mexico potash	#1	0·29	3·85	3·28	−15
New Mexico potash	#2	0·29	3·80	2·95	−22

by σ_z and P_0 which are the axial rock stress and radial rock stress respectively. E_{cpc} is from Table 2. N is the slope of the CPC pump up curve taken during measurements of E_{cpc} and is the change in volume (produced by the screw pump) required to cause unit pressure change. N varies with the actual fluid pressure; it decreases as the fluid pressure increases. The value of N shown in Table 3 is the one at the fluid pressure P_f also shown in the Table. The usual pattern of fluid pressure response is such that at low fluid pressures, the response is low, i.e. fluid pressures alter only slightly with changes in rock stress. As the fluid pressure increases, the gage becomes more responsive until a fairly constant response is obtained over a wide range of fluid pressures. These changes in response are paralleled by similar changes in the measured value of N. Thus, as Table 3 shows, the theoretical responses, as calculated using N, agree very well with the measured responses at all states of rock stress and fluid pressure.

One other thing remains to be explained and this is the factor K shown in Table 3. The factor K is necessary to account for the behavior of the copper shell of the CPC. Thus, during the initial pressure cycling of the CPC, actual fluid pressure changes P_f, are greater than the pressure changes on the inside of the borehole, P_t, since some of the fluid pressure is used in expanding the copper shell, i.e. $P_f = KP_t$, where $K > 1$. However, when the gage is responding to rock stress changes, then actual pressure changes on the inside of the borehole, P_{t_2}, are greater than the fluid pressure changes within the gage P_f, i.e. $P_{t_2} = KP_f$.

This means that the measured value of N requires a certain modification and analysis shows that the value of N in equation (11) must be replaced by $N + 2\pi La^2 (1+\nu)(K-1/K)/E$.

The value of K can be calculated from the equation $K = 1/[1 - tE_c(1+\nu)/aE]$ where t is the thickness of the copper shell. E_c is the Young's modulus of the copper, a is the radius of the borehole, and E and ν are the elastic moduli of the rock.

Another factor should be mentioned concerning the calculation of the theoretical response. In an infinite medium, equation (10) as modified above will suffice. In the laboratory it tests,

TABLE 3. CYLINDRICAL PRESSURE CELL RESPONSES TO ROCK STRESS CHANGES

Rock type		L (in.)	a (in.)	ν	K	E_{sg} ($\times 10^6$)	E_{CPC} ($\times 10^6$ psi)	σ_z	P_0 (psi)	P_f	N (in^3 $\times 10^6$ psi)	Measured response (%) $\Delta P_f/\Delta P_0$	Theoretical response (%) E_{sg}	Theoretical response (%) E_{CPC}	Theoretical response–measured response E_{sg}	Theoretical response–measured response E_{CPC}
Indiana limestone	#2	6·85	0·75	0·23	1·32	4·70	5·41	2200	2200	900	21·3	38	44·5	39·5	+6·5	+1·5
Indiana limestone	#2	6·85	0·75	0·23	1·32	4·70	5·41	2200	2200	2500	16·7	50	54·5	48·5	+4·5	+1·5
Pennsylvania limestone	#1	6·90	0·75	0·39	1·30	10·95	10·1	2200	2200	700	20·9	17	21	22·5	+4	+5·5
Pennsylvania limestone	#1	6·90	0·75	0·39	1·30	10·95	10·1	2200	2200	2300	15·1	25	28	30·5	+3	+5·5
Pennsylvania limestone	#2	7·0	0·74	0·42	1·38	10·25	9·2	2200	2200	600	17·9	21	25·5	28	+4·5	+7
Pennsylvania limestone	#2	7·0	0·74	0·42	1·38	10·25	9·2	2200	2200	2300	14·2	28	31	34	+3	+6
Kansas salt	#2	6·9	0·75	0·31	1·53	4·24	4·08	2200	2200	600	29·2	26	34·5	35	+8·5	+9
Kansas salt	#2	6·9	0·75	0·31	1·53	4·24	4·08	2200	2200	2000	18·6	42	49	50	+7	+8
Georgia granite	#1	6·8	0·75	0·20	1·2		7·11	4000	4000	2500	12·9	42		50		+8
Georgia granite	#1	6·8	0·75	0·20	1·2		4·47	2200	1000	2900	15·1	69		64·5		−3·5
Georgia granite	#2	6·9	0·75	0·20	1·18	6·66	6·12	2000	2000	2600	9·6	70	70·5	76	+0·5	+6
Georgia granite	#2	6·9	0·75	0·24	1·18	7·20	7·09	4000	4000	3000	9·2	62	68	68·5	+6	+6·5
Ohio limestone	#1	6·9	0·75	0·3	1·27	8·45	8·26	2000	2000	3100	12·3	50	45	45	−5·5	−5
Ohio limestone	#1	6·9	0·75	0·3	1·27	8·03	7·64	4000	4000	3900	12·5	39	45	47·5	+6	+8·5
Ohio limestone	#2	6·9	0·75	0·27	1·15	7·25	8·15	2000	2000	1200	15·6	39	43	38·5	+4	−0·5
Ohio limestone	#2	6·9	0·75	0·27	1·15	7·51	8·50	4000	4000	3900	13·5	44	47·5	42·5	+5	−1·5
Pennsylvania diabase	#1	6·8	0·75	0·30	1·17	15·77		4000	4000	3900	12·4	27	25·5		−1·5	
Pennsylvania diabase	#1	6·8	0·75	0·30	1·20	15·77	14·1	4000	4000	4000	7·8	38	38·5	45·5	+0·5	+7·5
Pennsylvania diabase	#2	6·9	0·75	0·26	1·26	14·45	13·2	4000	4000	2000	7·7	38	42	45	+4	+7
Ohio salt	#1	6·9	0·75	0·22	1·9	4·25	3·38	2200	2200	1400	16·1	49	48·5	55·5	−0·5	+6·5
Ohio salt	#2	6·8	0·75	0·24	1·76	3·84	4·27	2200	2200	2700	18·0	50	51·0	47·5	+1	−2·5
New Mexico potash	#1	6·75	0·77	0·29	1·31	3·85	3·28	2200	2200	2000	16·3	55	66·5	75	+11·5	+20
New Mexico potash	#2	6·75	0·77	0·29	1·29	3·80	2·95	2200	2200	2000	17·5	42·5	64·5	79	+22	+37

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was necessary to correct for the fact that the cylinders were of finite dimensions. This was done using the usual thick-walled cylinder formulae.

Table 3 shows the excellent agreement between measured and theoretical responses. For instance, the theoretical response using E calculated from the CPC tests was on the average 2% higher than the measured response with a standard deviation of $\pm 5\%$. Tests were also conducted to determine the response of the CPC to rock stress changes in a direction parallel to the axis of the borehole. Results were quite consistent and revealed that the response $\Delta P_f / \Delta \sigma_z$ was $-4 \cdot 5\%$.

It is interesting to note that CPC responses in the salts and potash behaved in the same highly predictable manner as occurred in the more elastic rocks. But it should be remarked that the salt and potash had perhaps undergone a certain amount of work hardening during loading cycles of previous tests.

After the CPC tests were finished, the central hole was enlarged to $2\frac{1}{4}$ in. diameter and a borehole pressure cell (BPC) of the flatjack type was grouted into the hole. As before, the BPC was pressurized while maintaining constant axial and radial pressures on the rock and the volume change required to produce unit pressure change, N, was measured. Following this, the response of the BPC fluid pressure to radial rock stress changes was measured and compared with the theoretical response calculated from equation (14). Table 4 shows results of the tests on five strong elastic rocks. The average theoretical response, as calculated by equation (14) was found to be 8% higher than the average of the actual measured responses with a standard deviation of $\pm 6\%$.

Equation (14) applies only to rock stress changes perpendicular to the plane of the BPC. In the tests, pressure changes were applied to the rock cylinder in a uniform radial manner. The BPC is also sensitive to rock stress changes parallel to the plane of the BPC and the Appendix shows how this cross-sensitivity might be calculated using the theory of inclusions. Two types of grout were used around the BPC's, one was Portland Cement ($E = 7 \times 10^6$ psi, $\nu = 0 \cdot 18$) and the other was Lumnite Cement ($E = 9 \cdot 65 \times 10^6$ psi, $\nu = 0 \cdot 17$). Table 4 shows the theoretical cross-sensitivity calculated for both plane stress and plane strain. Actually, the condition of axial loading in the rock cylinders was neither plane strain nor plane stress. As an approximation, the theoretical cross-sensitivity was taken to be the average of the plane stress and plane strain conditions. When this theoretical cross-sensitivity was added to the theoretical response of equation (14), then the means of the theoretical and measured responses for 10 rock samples agreed within 1% with a standard deviation of $\pm 6\%$.

Tests were conducted to determine the response of the BPC to rock stress changes in a direction parallel to the borehole axis. The measured response was extremely small in both strong elastic rocks and in weak plastic rocks. The highest measured response in the eight rock types tested was $\pm 2 \cdot 5\%$, and the average of the eight rock types tested was 0% with a standard deviation of $\pm 1\%$.

BPC responses to stress changes in plastic rocks showed erratic behavior. The results shown in Table 5 reveal that in three rocks, the BPC's behaved in a predictable manner, whereas, in three others the theoretically calculated responses were much higher than the measured responses. The behavior was predictable only when the BPC fluid pressure was close to or higher than the rock stress. When the BPC fluid pressure was appreciably lower than the stress in the rock, it was observed that the measured values of N were very low.

All the tests discussed here were short-term tests. It would have been most instructive to conduct similar tests in plastic material to see if the BPC fluid pressure and the CPC fluid

TABLE 4. BOREHOLE PRESSURE CELL RESPONSE TO ROCK STRESS CHANGES IN STRONG ELASTIC ROCKS

Rock type		Mortar	E_{sg} ($\times 10^6$psi)	σ_2 (psi)	P_0 (psi)	P_f (psi)	N^3 (in^3/psi $\times 10^{-6}$)	Theoretical response $\Delta P_f/\Delta P_0$ (%)	Measured response $\Delta P_f/\Delta P_0$ (%)	Difference theoretical measured (%)	Theoretical cross-sensitivity Plane stress (%)	Theoretical cross-sensitivity Plane strain (%)
Indiana limestone	#1	L	4·98	1200	2200	1600	17·1	75	64	+11	−26	−4
Indiana limestone	#2	P	4·70	1200	2200	1600	15·7	87	64	+23	−13	−2
Pennsylvania limestone	#1	L	10·95	1200	2200	1600	10·2	57·5	43	+14·5	−14	−13
Pennsylvania limestone	#2	P	10·25	1200	2200	1600	11·5	54·5	42	+12·5	−10	−13
Ohio limestone	#1	P	8·45	2000	2000	1400	13·6	55·5	50	+5·5	−6	−7
Ohio limestone	#2	L	7·51	2000	2000	1400	18·7	45·5	39·5	+6	−14	−6
Pennsylvania diabase	#1	L	15·77	2000	2000	1400	13·1	31	30·5	+0·5	−2	−7
Pennsylvania diabase	#2	P	14·45	2000	2000	1400	11·9	37	31·5	+5·5	+4	−4
Georgia granite	#1	P	4·58	1200	1200	1200	20·6	68	60	+8	−14	−2
Georgia granite	#1	P	5·88	2400	2400	3000	13·4	81·5	69	+12·5	−6	−1
Georgia granite	#2	L	6·66	2000	2000	1200	18·5	52	42·5	+9·5	−8	+2
Georgia granite	#2	L	7·20	4000	4000	3100	13·5	66	47·5	+18·5	−8·5	−3·5

TABLE 5. BOREHOLE PRESSURE CELL RESPONSE TO ROCK STRESS CHANGES IN PLASTIC ROCKS

Rock type		E_{sg} ($\times 10^6$psi)	σ_z (psi)	P_0 (psi)	P_f (psi)	in^3/psi $\times 10^{-6}$	Theoretical response $\Delta P_f/\Delta P_0$ (%)	Measured response $\Delta P_f/\Delta P_0$ (%)	Difference theoretical measured (%)	Theoretical cross-sensitivity Plane stress (%)	Theoretical cross-sensitivity Plane strain (%)
Kansas salt	#1	4·14	1200	2200	1600	16·6	94	60	+33·5	−29	−11
Kansas salt	#2	4·24	1200	2200	1500	16·8	90	60	+30	−25	−9
Ohio salt	#1	4·25	1200	2200	1800	16·0	94	68	+26	−16	−1
Ohio salt	#2	3·84	1200	1200	1500	30·3	55	60	−5	−33	−2
Ohio salt	#2	3·84	1200	2200	2200	21·1	79	70	+9	−33	−2
New Mexico potash	#1	3·80	1200	1200	1500	29·8	57	60	−3	−37	−7
New Mexico potash	#1	3·80	1200	2200	2100	21·1	79	75	+4	−37	−7
New Mexico potash	#2	3·80	1200	1200	1400	22·1	76·5	70	+6·5	−27	−7
New Mexico potash	#2	3·80	1200	2200	2100	18·2	93	76	+17	−27	−7

pressure would stabilize at pressures similar to the rock stresses after long periods of plastic flow in the rocks. The tests here did show that such rocks as salt and potash did have an elastic type behavior which was predictable.

CONCLUSIONS

The following procedure should be adopted to convert observed fluid pressure change in a hydraulic borehole gage to changes of rock stress in the plane perpendicular to the axis of the borehole in which the instrument is pressured.

(1) The Young's modulus and Poisson's ratio of the rock surrounding the gage must be determined. This may be done by testing a prepared sample or by using a CPC to find the modulus of rigidity. If the latter method is used, it will be necessary to assume a value of Poisson's ratio.

(2) The 'stiffness' of the hydraulic borehole gage must be measured. A screw pump should be used, first to measure the pressure/volume relationship, N_t, of the entire hydraulic cell—2 Bourdon gages—screw pump system and secondly, to measure the pressure volume relationship, N_{pump} of the screw pump and one Bourdon gage alone. The screw pump and one Bourdon gage should then be detached from the hydraulic cell system whose stiffness will be given by $N = N_t - N_{\text{pump}}$.
It is recommended that N be measured on an ascending fluid pressure cycle after sufficient preliminary cycles have been performed to establish an elastic behavior of both rock and gage. Generally, the response of both CPC's and BPC's depends on the fluid pressure but this would automatically be taken into account by using the value of N measured at the observed fluid pressure. The response of a gage can be improved by increasing its stiffness. This can be done by reducing the fluid volume to a minimum and by using a photoelastic gage instead of a Bourdon tube gage [4].

(3) In the case of a CPC, the copper shell factor, K, must be determined. This can be done using the equation $K = 1/[1 - tE_c(1+\nu)/aE]$.

(4) The conversion of fluid pressures, P_f, to the mutually perpendicular rock stresses S and T acting in the plane whose normal lies in a direction parallel to the axis of the borehole, for the case of the CPC, may be performed using the equation

$$\frac{P_f}{(S+T)} = \frac{2\pi La^2}{E}\left[\frac{1-\nu^2}{N + \dfrac{2\pi La^2}{E}(1+\nu)(K - 1/K)}\right].$$

(5) In the case of a flatjack type cell, the observed fluid pressure change can be converted to a rock stress change, S, in a direction normal to the plane of the flatjack. However, a slight correction must be made for any rock stress change T acting in a direction perpendicular to S and to the axis of the borehole. This correction may be made on the basis of the theory of stresses and strains around a heterogeneous inclusion under conditions of plane strain if the elastic constants of the inclusion can be assumed to be those of the mortar and if the elastic constants of the mortar are known. Since this method of correction involves much additional work and is of doubtful accuracy anyway, it is suggested that the cross-sensitivity of a flatjack, when used in a strong elastic rock, be taken as -6% of the rock stress change T. The

conversion of flatjack fluid pressure changes, P_f, to rock stress changes S may then be performed using the equation

$$\frac{P_f}{S - 0{\cdot}06T} = \frac{2\pi La^2(1-\nu^2)}{NE}.$$

(6) The response of a borehole flatjack to changes of stress $\Delta\sigma_z$, in a direction parallel to the axis of the borehole can be ignored. For a CPC the response is small ($P_f/\Delta\sigma_z = -0{\cdot}045$) and unless $\Delta\sigma_z$ is known to be large, its effect can be ignored.

(7) Where the rock material exhibits plastic flow, the conversion of borehole flatjack fluid pressure changes to rock stress changes can be performed in the same manner as that for the stronger elastic rocks, provided that the flatjack fluid pressures are maintained at a level equal to or greater than the inherent rock stress. However, the influence of the plastic flow makes the accuracy of the conversion uncertain, and there is reason to believe that after a period of time the flatjack fluid pressure or CPC fluid pressure will stabilize to a value close to the rock stress.

(8) In order to obtain as complete a picture of the rock stress changes as possible, two borehole flatjacks should be installed adjacent to each other and oriented at right angles to each other in the same borehole. If a CPC is used to measure rock stress changes, the installation of a borehole flatjack close by would permit the sum of the rock stress changes S and T, as measured by the CPC, to be resolved into its components.

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APPENDIX

Plane stress

Stress distributions in and around a circular inclusion in a two-dimensional stress field have been solved by Sezawa and Nishimura [8] and [9]. In this paper, we are interested only in the stresses inside the inclusion, and analysis shows that:

$$\sigma_{r_2} = \frac{S}{2}(D + A\cos 2\theta) + \frac{T}{2}(D - A\cos 2\theta) \tag{16}$$

$$\sigma_{\theta_2} = \frac{S}{2}(D - A\cos 2\theta) + \frac{T}{2}(D + A\cos 2\theta). \tag{17}$$

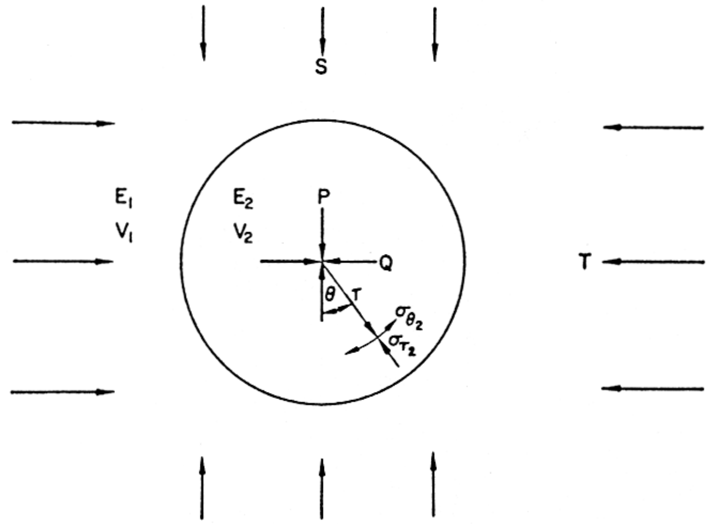

FIG. 3. Stresses in and around a circular inclusion in a two-dimensional stress field.

Note that both σ_{r_2} and σ_{θ_2} are independent of r which means that the principal stress P and Q are constant throughout the inclusion, both in magnitude and direction.

In the above equations

$$D = \frac{(\lambda_2 + \mu_2)(\lambda_1 + 2\mu_1)}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2 + \mu_1)}$$

and

$$A = \frac{4\mu_2}{\mu_2 + 3\mu_1}$$

where

$$\lambda_1 = \frac{\nu_1 E_1}{(1 + \nu_1)(1 - 2\nu_1)}$$

$$\mu_1 = \frac{E_1}{2(1 + \nu_1)}$$

and similarly for λ_2 and μ_2.

E_1 and ν_1 are the Young's modulus and Poisson's ratio of the material surrounding the inclusion. E_2 and ν_2 are the elastic moduli of the inclusion material.

Principal stresses in the inclusion will be:

$$P = S(D + A)/2 + T(D - A)/2 \tag{18}$$

$$Q = S(D - A)/2 + T(D + A)/2. \tag{19}$$

In the particular case of a borehole flatjack whose plane lies perpendicular to the direction of S, then the fluid pressure of the BPC will be equal to P, and the response of the BPC to stress T will be $\Delta P_f = \Delta T(D-A)/2$.

Plane strain

The case of a cylindrical inclusion in a biaxial stress field acting at infinity under conditions of plane strain has been solved by GOODIER [10]. Using the same notation as before, analysis shows that the stresses inside the inclusion are given by the equations:

$$\sigma_{r_2} = 2\mu_2(1 - \nu_1)\left[\frac{S + T}{2[(1 - 2\nu_2)\mu_1 + \mu_2]} + \frac{(S - T)\cos 2\theta}{\mu_1 + (3 - 4\nu_1)\mu_2}\right] \tag{20}$$

$$\sigma_{\theta_2} = 2\mu_2(1 - \nu_1)\left[\frac{S + T}{2[(1 - 2\nu_2)\mu_1 + \mu_2]} - \frac{(S - T)\cos 2\theta}{\mu_1 + (3 - 4\nu_1)\mu_2}\right] \tag{21}$$

and again, the principal stresses are obtained by substituting $\theta = 0°$. Note that the first term in these two equations is identical to the term $D/2$ in equations (16) and (17), from which it follows that when $S = T$, the plane stress and plane strain solutions are identical.

If a BPC is oriented so that its plane lies perpendicular to S, then the fluid pressure change, P_f, in the BPC, will be equal to P and the response of the BPC to stress T will be

$$\Delta P_f = \Delta T \left[\frac{\mu_2 (1 - \nu_1)}{(1 - 2\nu_2)\,\mu_1 + \mu_2} - \frac{2\mu_2 (1 - \nu_1)}{\mu_1 + (3 - 4\nu_1)\,\mu_2} \right].$$

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